

S4MA Set D Paper 2  
www.pmc.sg

# **Sec 4 Maths**

**Exam papers with worked solutions**

## **SET D PAPER 2**

Compiled by

**THE MATHS CAFE**

S4MA Set D Paper 1  
www.pmc.sg

Answer **all** questions.

Write your answers and working on the separate Answer Paper provided.

Give non-exact numerical answer correct to 3 significant figures, or 1 decimal place in case of angles in degrees, unless a different level of accuracy is specified in the questions.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks in this paper is 100.

TheMathsGate

**1. ALGEBRA***Quadratic Equation*For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

*Formulae for  $\Delta ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Prove the identity

$$\frac{1 - \sin 2A}{1 + \cos 2A} = \frac{1}{2}(1 + \tan A)^2 \quad [4]$$

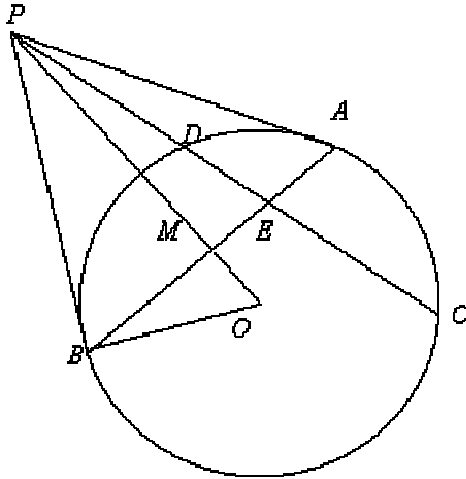
2 Express  $\frac{5x+14}{(2x-1)(3x+4)}$  as partial fractions and hence,

$$\text{find } \int_1^4 \frac{5x+14}{(2x-1)(3x+4)} dx \quad [6]$$

S4MA Set D Paper 1  
www.pmc.sg

- 3 a) Solve the equation  $9^x - 4 = 3^{x+1}$ . [5]
- b) The equation  $2x^2 - 9x + 4 = 0$  has roots  $\alpha$  and  $\beta$  form a quadratic equation in  $x$ , whose roots are  $\alpha^2 + 2$  and  $\beta^2 + 2$ . [6]

- 4 PA and PB are two tangents to the circle with centre O. PO and BA meet at M, PC and BA meet at the point E.



Prove that

- (a)  $\triangle PMA$  and  $\triangle BMO$  are similar, [3]  
(b)  $PA \times BM = BO \times PM$  , [2]  
(c)  $PE^2 = PD \times PC - DE \times CE$  [4].

5 Given that the coefficient of  $x^4$  in the expansion of  $\left(x^3 - \frac{m}{x}\right)^8$  is  $-13608$ ,

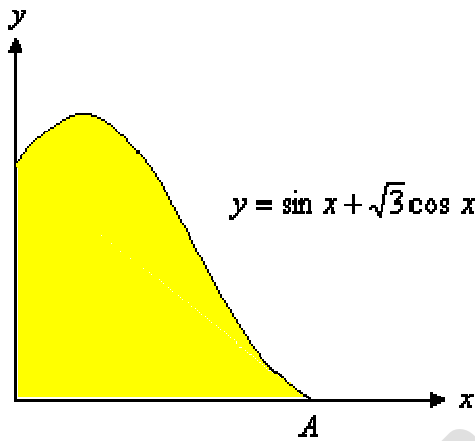
(i) find the value of the constant  $m$ , [3]

(ii) hence find the term independent of  $x$  in the expansion

of  $\left(1 + \frac{2}{x^4}\right)\left(x^3 - \frac{m}{x}\right)^8$ . [5]

6 a) Evaluate  $\int_0^{\frac{\pi}{4}} (\sec^2 x - \sin 4x) dx$  [4]

b)



The diagram shows part of the graph of  $y = \sin x + \sqrt{3} \cos x$ . The graph intersects the  $x$ -axis at point  $A$ .

- (i) Show that the  $x$ -coordinate of  $A$  is  $\frac{2\pi}{3}$ . [4]
- (ii) Calculate the area of the shaded region. [3]

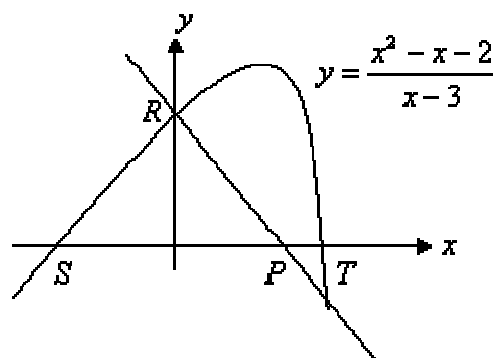


- 7 (a) Solve the equation  $|x^2 + x - 6| = 6$ . [3]
- (b) Sketch the graph of  $y = |x^2 + x - 6|$ . [3]
- (c) Hence state the range of values of  $x$  for which  $|x^2 + x - 6| < 6$ . [1]

- 8 The equation of a curve is  $y = x^2 e^{2x}$ .
- Find the stationary points and determine the nature of each stationary point. [7]

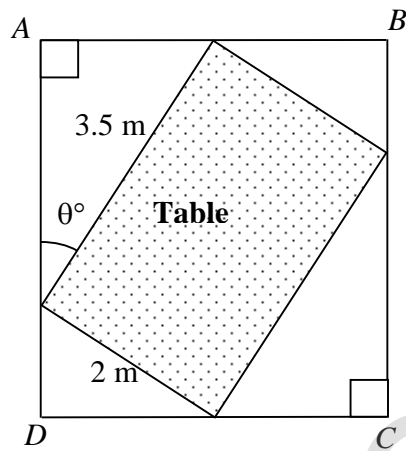
- 9 The line  $2y + x = 12$  cuts the circle  $C$  with equation  $x^2 + y^2 - 6x - 4y - 12 = 0$  at two points  $A$  and  $B$  where  $A$  is on the  $y$ -axis.
- (i) Find the coordinates of the centre and the radius of circle  $C$ . [2]
  - (ii) Find the coordinates of  $A$  and  $B$ . [2]
  - (iii) Find the length  $AB$ . [1]
  - (iv) Given that  $D$  is a point on the circle such that the line segment  $AD$  is a diameter of the circle, find the equation of the line  $BD$ . [3]

TheMathsGate



The diagram shows part of the graph of the curve  $y = \frac{x^2 - x - 2}{x - 3}$ .

- (a) Given that  $\frac{dy}{dx} = 1 + \frac{k}{(x-3)^2}$ . Find the value of  $k$ . [4]
- (b)  $R$ ,  $S$  and  $T$  are intersections of the graph with the axes.  $PR$  is the normal to the curve at  $R$ . Find the equation of  $PR$ . [3]
- (c) Find the ratio of the area of triangle  $PSR$  to triangle  $PTR$ . [4]



The diagram shows a rectangular table which is placed on a carpet  $ABCD$ .  
The dimensions of the table are 3.5 m by 2 m. The perimeter of the carpet is  $P$  m.

- (a) Show that  $P = 11\cos\theta^\circ + 11\sin\theta^\circ$ . [3]
- (b) Express  $P$  in the form  $R\cos(\theta^\circ - \alpha^\circ)$ . [2]
- (c) Find the maximum value of  $P$  and the corresponding value of  $\theta$ . [2]

S4MA Set D Paper 1

www.pmc.sg

12 A particle moving in a straight line passes a fixed point  $O$  with a velocity of 15 m/s. Its acceleration,  $a$  m/s<sup>2</sup> is given by  $a = 7 - 4t$ , where  $t$  is the time in seconds after passing  $O$ . Find

- (i) the maximum speed reached by the particle in the original direction of the motion, [3]
- (ii) the value of  $t$  when the particle is instantaneous at rest, [2]
- (iii) the displacement of the particle from  $O$  when  $t = 6$  seconds, [3]
- (iv) the average speed of the particle during the first 6 seconds. [3]