Sec 4 Maths

Exam papers with worked solutions

SET D
PAPER 1

Compiled by

THE MATHS CAFE

Write your name, class and index number on every answer paper.

Answer all the questions.

Write your answers and working on the separate answer paper.

If you use more than one sheet of paper, fasten the sheets together.

Give non-exact numerical numbers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets[ ] at the end of each question or part question.

The total marks for this paper is 80.

The use of electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}.$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}bc\sin A.$$

1 The expression $2x^3 + ax^2 + 3x + b$ has a factor $(x - 1)$ and leaves a remainder –54 when divided by $(x + 2)$. Find the values of a and b. [4]

2 (a) Simplify $\dfrac{16^{x+1} + 20\left(4^{2x}\right)}{2^{x-3}\,8^{x+2}}$ [2]

(b) Without using a calculator, solve the equation

$$\sqrt{27^{2x+3}} = \frac{1}{3^{x-2} \times 9^{3x}}$$ [3]

3 In triangle PQR, PQ = $\frac{1}{\sqrt{3}-1}$ *cm*, QR = $2\sqrt{2}$ *cm* and $\angle$ PQR = 90^o.
A square is formed with PR as one side of the square. Find the area of the square in the form $\left(a\sqrt{3}+b\right)$. [5]

4 (a) Solve the equation $2 \lg x + \lg 4 = \lg (9x - 2)$. [3]

(b) Solve the equation $\log_8 \frac{2}{\sqrt{x}} - 1 = \log_2 \sqrt{x}$. [3]

5 Given that $y = 27x^3 + \frac{1}{x} + 1$,

(i) write down an expression for $\frac{dy}{dx}$,

(ii) find the x coordinates of the stationary points,

(iii) determine the nature of the stationary points.

[6]

6 (a) Solve the inequality $(2x - 1)(3 - x) \leq -12$. [3]

(b) Find the range of values of m for which $x^2 + (m+3)x + 2m + 3 > 0$ for all real values of x.

[3]

7 A straight line through the point C (6, 0) meets the curve $y^2 = 4x$ at A (4, 4). Find

(a) the coordinates of the point B at which the line meets the curve again,

(b) the ratio of AC : CB.

[6]

8 (a) Prove the identity

$$\frac{1-2\sin x\cos x}{\sin^2 x-\cos^2 x}=\frac{\tan x-1}{\tan x+1}$$ [3]

(b) Solve the equation for values of θ between 0 and 2π,

$$2\cos\theta=\sqrt{3}\cot\theta$$

leaving your answers in terms of π. [4]

9 Sketch on the same diagram the graphs of $y_1 = 2\sin\frac{x}{2}$ and $y_2 = 1+\cos 2x$ for the interval $0 \le x \le 2\pi$.

State the amplitudes and periods of y_1 and y_2. Label each graph clearly.

Hence, state the number of solutions in this interval for the solutions of the equation $2\sin\frac{x}{2} - \cos 2x = 1$.

[8]

10 (a) Given $y = x^3 \ln x$, show that $\frac{dy}{dx} = 3x^2 \ln x + x^2$.

Hence, evaluate $\int_1^2 x^2 \ln x \, dx$.

[4]

(b) Given that $e^y = 6x^2 - 5$, find $\frac{dy}{dx}$.

Hence, find the rate of change of y at $x = 1.5$, given that x is increasing at the rate of 2 units per second.

[4]

11 (a) The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	8.7	15.0	25.0	45.0	77.9

It is known that the variables x and y are related by the equation

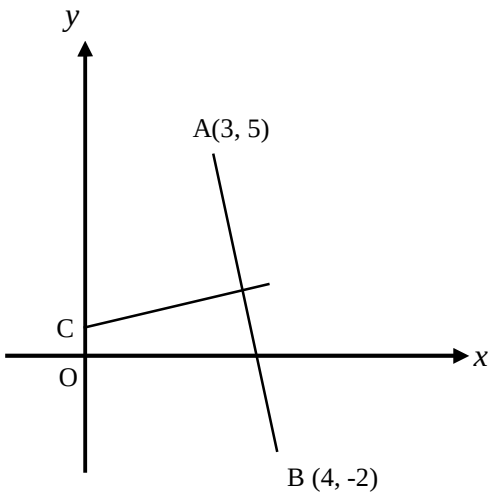
$y = ab^{0.5x}$ where a and b are constants.

(i) By plotting the graph of lg y against x, obtain a straight line graph to represent the above data. [3]

(ii) Use the graph to estimate the value of a and of b. [3]

(b) The variables x and y are related in such a way that when $(y - x)$ is plotted against x^3, a straight line is obtained which passes through (1, 5.5) and (8, 9). Find y in terms of x. [3]

12 **Solutions to this question by accurate drawing will not be accepted.**



A and B are the points (3, 5) and (4, –2) respectively.

(i) Find the equation of the perpendicular bisector of AB. [3]

(ii) Find the coordinates of C, the point where the perpendicular bisector cuts the *y*–axis. [1]

(iii) Show that ACBD is a square if the coordinates of D are (7, 2). [3]

(iv) Find the coordinates of mid-point of BC. [1]

Hence find the coordinates of E if ACEB is a parallelogram. [2]

Answer keys:

1 $a = -9,\ b = 4$

2 (a) $\frac{9}{2}$ (b) $x = -\frac{1}{4}$

3 Area of the square = $PR^2 = \frac{1}{2}\sqrt{3} + 9$

4 (a) $x = \frac{1}{4}$ or $x = 2$ (b) $x = \frac{1}{2}$

5 (i) $\frac{dy}{dx} = 81x^2 - \frac{1}{x^2}$ (ii) $x = \frac{1}{3}$ or $x = -\frac{1}{3}$

(iii) the point at $x = \frac{1}{3}$ is a min point, the point at $x = -\frac{1}{3}$ is a max point.

6 (a) $x \le -1$ or $x \ge 4\frac{1}{2}$ (b) $-1 \le m \le 3$

7 (a) B(9, –6) (b) 2 : 3

8 (b) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

9 y_1 : amplitude = 2 , period = 4π

y_2 : amplitude = 1, period = π

Number of solutions = 3

10 (a) 1.07 (b) $\frac{dy}{dx} = 2.1176 = 2.12$, $\frac{dy}{dx} = 4.24$

11 (a)

x	1	2	3	4	5
lg y	0.94	1.18	1.40	1.65	1.59

$a = 5.01$ $b = 2.99$

(b) $y = \frac{1}{2}x^3 + x + 5$

12 (i) $7y = x + 7$ (ii) C(0, 1) (iii) AC=CB=BD=DA=5 & AC $\perp$ CB

(iv) mid of BC = (1, – 6), E (1, –6)