

Sec 4 Maths

Exam papers with worked solutions

SET C

PAPER 2

Question

Compiled by

THE MATHS CAFE

S4MA Set C Paper 2 Question

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Answer **all** the questions.

1. (a) Solve the following equations.

(i) $e^x = 2e^{\frac{x}{2}} + 15$, using the substitution, $p = e^x$, [4]

(ii) $(2^{3x+2})(3^{x-1}) = 16$. [5]

(b) Variables x and y are connected by the relationship, $\log_3 y = p \log_3 x + q$, where p and q are constants. Given that $y = 27$ when $x = 3$ and $\log_3 y = 5$ when $x = 27$,

(i) find the value of p and q , [3]

(ii) express the relationship in the form $y = kx$. [3]

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2. (a) Solve the following equation for $0^\circ \leq x \leq 360^\circ$,

$$2(\tan x + 2) = \frac{1}{\cos^2 x}. \quad [4]$$

- (b) The function f is defined, for $0 \leq x \leq 2\pi$, by

$$f(x) = |3 \cos 2x|.$$

- (i) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. [3]

- (ii) Using the same axes, sketch a suitable graph and state the number of solutions to the equation $2 - |3 \cos 2x| = \frac{x}{2\pi}$. [3]

3. (a) The straight line $2x + 3y = 6$ intersects the curve $(2x + 1)^2 + 6(y - 2)^2 = 49$ at the points A and B . Calculate the distance of AB . [5]
- (b) Without using calculators, evaluate $\frac{1}{(3 - \sqrt{5})^2} + \frac{1}{(3 + \sqrt{5})^2}$. [4]

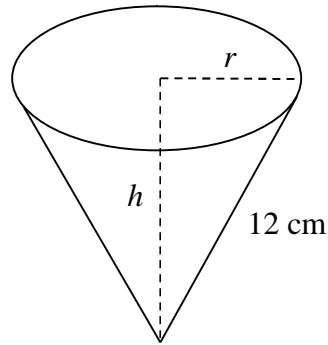
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4. Express $\frac{x}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2}$, where A and B are constants to be determined.

Hence, find $\int_3^4 \frac{x}{(x-2)^2} dx$.

[8]

5. The figure shows a right circular cone of depth h cm, radius r cm and slant height 12 cm.



- (i) Show that the volume of the cone, V cm³, is given by $V = 48\pi h - \frac{1}{3}\pi h^3$. [3]
- (ii) Find the value of h for which V is stationary and determine the nature of the stationary value. [4]

6. (a) Find

(i) $\int 8e^{\frac{x}{2}} dx$ [3]

(ii) $\int_{\frac{\pi}{4}}^{\pi} 6 \cos 2x \, dx.$ [3]

(b) (i) Differentiate xe^{2x} with respect to x . [2]

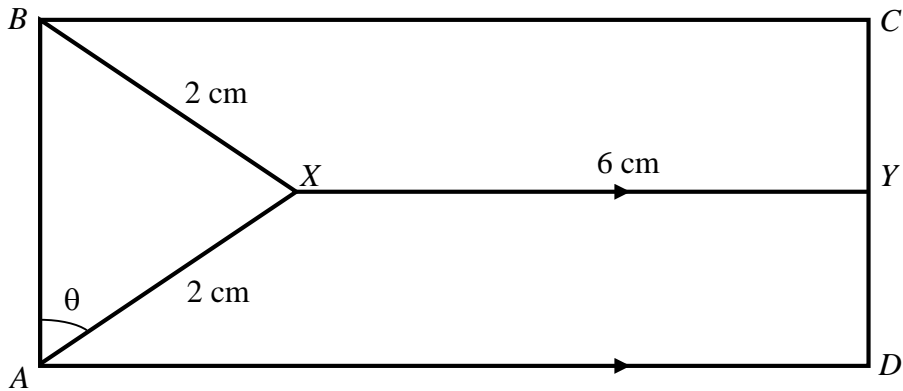
(ii) Find the x -coordinate of the stationary point of the curve $y = xe^{2x}$. [2]

(iii) Hence, show that $\int 4xe^{2x} dx = 2xe^{2x} - e^{2x} + c$, where c is a constant. [4]

7. (a) Find the range of values of k given that the function $2x^2 + 4x + 2 = k(x + 3)$ has two distinct roots. [4]
- (b) Given that α and β are the roots of the quadratic equation $2x^2 + 4x = 3$, form a quadratic equation whose roots are $2\alpha + 3$ and $2\beta + 3$. [4]

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8. The figure shows a rectangle $ABCD$. XAB is an isosceles triangle with $AX = BX = 2$ cm and $\angle XAB = \theta$ radians. XY is parallel to AD , and $XY = 6$ cm.



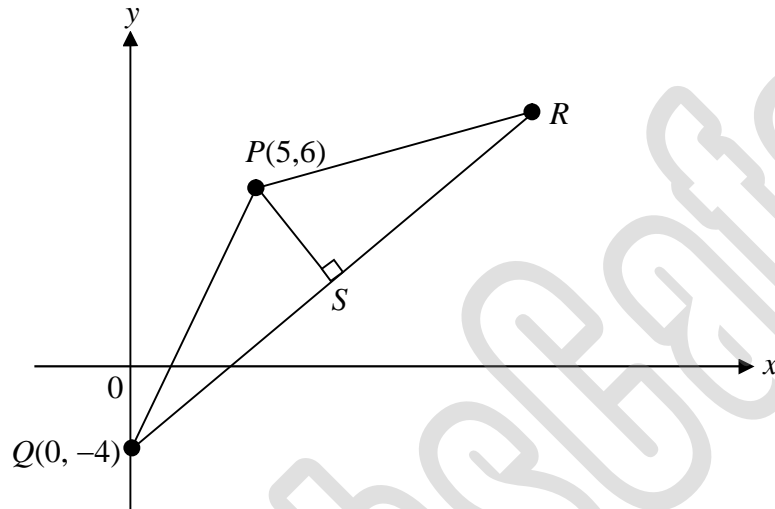
- (a) Show that the perimeter of the rectangle $ABCD$ is given by

$$P = 4 \sin \theta + 8 \cos \theta + 12. \quad [2]$$

- (b) Express P in the form $a + b \sin(\theta + \alpha)$, where $b > 0$ and $0 < \alpha < \frac{\pi}{2}$. [2]
- (c) Find the maximum value of P . [1]
- (d) Find the value of θ for which $P = 18$. [2]
- (e) Given that the area of rectangle $ABCD$ is larger than 5 times the area of triangle XAB by 6 cm^2 , show that $4 \cos \theta = \sin 2\theta + 1$. [3]

9. **Solution to this question by graph paper will not be accepted.**

The figure shows an isosceles triangle PQR in which $PQ = PR$. The coordinates of points P and Q are $(5, 6)$ and $(0, -4)$ respectively. The gradient of $QR = \frac{3}{4}$ and the perpendicular line from P to QR meets QR at point S .



Find

- (a) the equations of QR and PS , [3]
- (b) the coordinates of point R , [4]
- (c) the area of triangle PQR . [2]

10. **Answer the whole of this question on a piece of graph paper.**

The table below shows the corresponding values of two variables, x and y , that are related by the equation $y = kh^{-2x}$, where k and h are constants.

x	31.6	19.9	12.6	7.9	5.0	1.6
y	2	4	6	8	10	12

- (a) Show how a straight line of $\lg y$ against x can be drawn to represent the given equation, indicating clearly its gradient and vertical intercept. [2]
- (b) Plot the graph of $\lg y$ against x and use it to find the values of k and h . [5]
- (c) Using your graph, find the value of
- (i) x when $\lg y = 0.5$, [1]
- (ii) y when $x = 15$. [2]