

Sec 4 Maths

Exam papers with worked solutions

SET C

PAPER 1

Question

Compiled by

THE MATHS CAFE

S4MA Set C Paper 1 Question

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Answer **all** the questions.

1. Find all the angles, between 0° and 360° inclusive which satisfy the equations,

(a) $3 \sin x = 4 \sin x \tan x$ [3]

(b) $2 \operatorname{cosec} \left(\frac{3}{2}y - 80^\circ \right) = -4$ [3]

2. (a) Given that $\frac{\sin(A+B)}{\sin(A-B)} = \frac{7}{5}$, show that $\tan A = 6 \tan B$. [3]

(b) Using the result above and given that A is an acute angle and $\tan B = \frac{1}{3}$,
find
(i) $\tan(A-B)$, [2]
(ii) $\cos 2A$. [2]

3. (i) Differentiate $\tan^3 x$ with respect to x and express the derivative in terms of $\sec x$ only. [2]
- (ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$. [4]

4. Find the range of values of k for which the line $y = 3x - 4$ does not intersect the curve $y = 2x^2 - x + k$. [4]

5. The function $f(x) = 4x^4 + px^3 - 19x^2 + qx - 6$ has a factor of $(x + 2)$ and a stationary value at $x = \frac{1}{2}$.
- (i) Find the values of p and q . [5]
- (ii) Hence, find the remainder when $f(x)$ is divided by $(x - 4)$. [1]

6. Prove the identity $\tan^2 x + \cot^2 x = \frac{\sin^2 x + 2 \cos^4 x - \cos^2 x}{\cos^2 x \sin^2 x}$. [5]

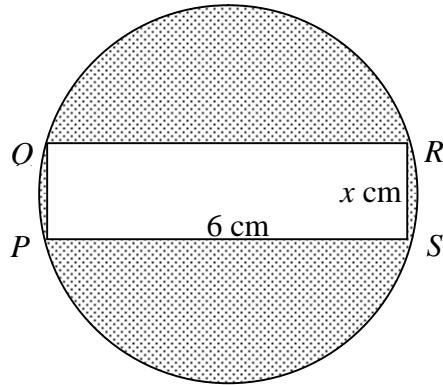
7. Solve the equation $\log_{16}(3x-1) = \log_4(3x) + \log_4(0.5)$. [6]

8. (i) Find the coefficient of x^{21} in the expansion of $(4 - x^3)^{10}$. [4]

(ii) Hence, find the coefficient of x^{22} in the expansion of $(1+2x)(4 - x^3)^{10}$. [2]

9. (a) Given that $y = 4x(3 - x)$,
- (i) find $\frac{dy}{dx}$ and determine the nature of the stationary point, and [3]
- (ii) hence, sketch the graph of $y = 4x(3 - x)$. [1]
- (b) The volume, $V \text{ cm}^3$, of liquid in a container is given by $V = 2x^3 - 3x^2 + 8$, where $x \text{ cm}$ is the depth of the liquid in the container. The liquid is poured into the container at a rate of $180 \text{ cm}^3/\text{s}$. Find the rate of change of the depth, in cm/s , of the liquid at the instant when its depth is 5 cm . [4]

10. The diagram shows a rectangle $PQRS$ inscribed in a circle. $PS = 6$ cm and $RS = x$ cm.



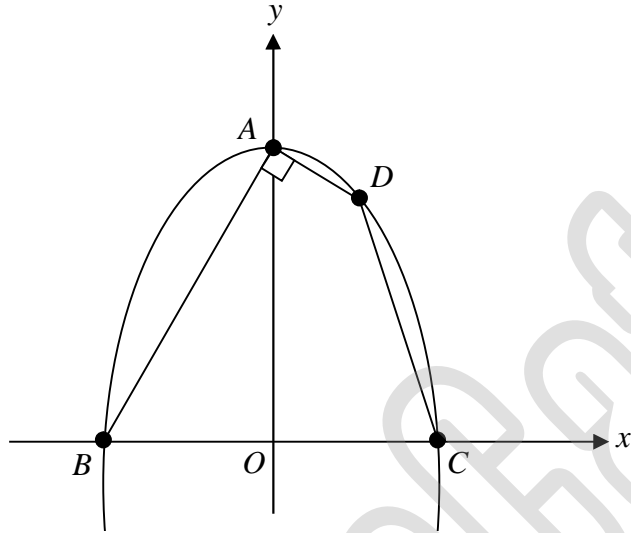
- (i) Show that the area, A cm², of the shaded region, is given by

$$A = \frac{\pi}{4}x^2 - 6x + 9\pi . \quad [4]$$

- (ii) Calculate the value of x in terms of π when the area of the shaded region is a minimum. [4]

11. **Solutions to this question by accurate drawing on graph paper will not be accepted.**

The figure shows a curve $y = -x^2 + 4$ which intersects the x -axis at Point B and C and the y -axis at point A . D is a point on the curve such that AD is perpendicular to AB .



Find

- (a) the equation of AB , [4]
(b) the coordinates of point D , [4]
(c) the equation of the line through B which is parallel to AD , [2]
(d) the area of quadrilateral $BADC$. [2]

12. The equation of a circle is $x^2 + y^2 - 6x - 8y + 16 = 0$.
- (a) Find the radius and the coordinates of the centre of the circle. [2]
- (b) Show that the circle touches the line $y = 7$. [1]
- (c) Find the equation of the circle which is a reflection of the circle, $x^2 + y^2 - 6x - 8y + 16 = 0$ in the line $y = 7$. [3]