Sec 4 Maths

Exam papers with worked solutions

SET B
PAPER 2
Question

Compiled by

THE MATHS CAFE

1. It is given that $t = \frac{2}{\sqrt{3}}$ and $y = \frac{2t+1}{t-1}$. Express each of the following in its simplest surd form:

(i) y, [2]

(ii) $y - \frac{26}{y}$. [2]

2. Express $\frac{3x^2 - 4x - 1}{(x-2)(x^2+1)}$ into partial fractions. Hence evaluate $\int_3^5 \frac{3x^2 - 4x + 1}{(x-2)(x^2+1)}\,dx$. [5]

3. The roots of the equation $2x^2 - 9x - 7 = 0$ are $2\beta + \alpha$ and $2\alpha + \beta$.

(i) Find the value of $\alpha + \beta$ and of $\alpha\beta$. [4]

(ii) If the roots of the equation $x^2 + px + q = 0$, where p and q are constants, are α and β, find the value of p and of q. [2]

4. The curve $y = \ln(3 + 2x)$ meets the x-axis at P and y-axis at Q.

(i) Find the coordinates of P and of Q. [2]

(ii) Sketch the curve, indicating clearly the axes intercepts and the asymptote. [3]

(iii) Find the equation of the straight line which must be drawn on the graph of $y = \ln(3 + 2x)$ to obtain the solution to the equation $3e^{2x} + 2xe^{2x} = \frac{1}{e^2}$. [2]

5. **(a)** In the expansion of $(2+5x)^n$ where n is a positive integer, the coefficients of x^3 and x^4 are in the ratio $8 : 15$ Find the value of n. [3]

(b) Find the term independent of x in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$. [4]

6. A curve passes through the point $\left(\frac{\pi}{2}, 0\right)$ and its gradient at any point (x, y) on the curve is $\sin 2x\,(1 - \sin 2x)$. Find

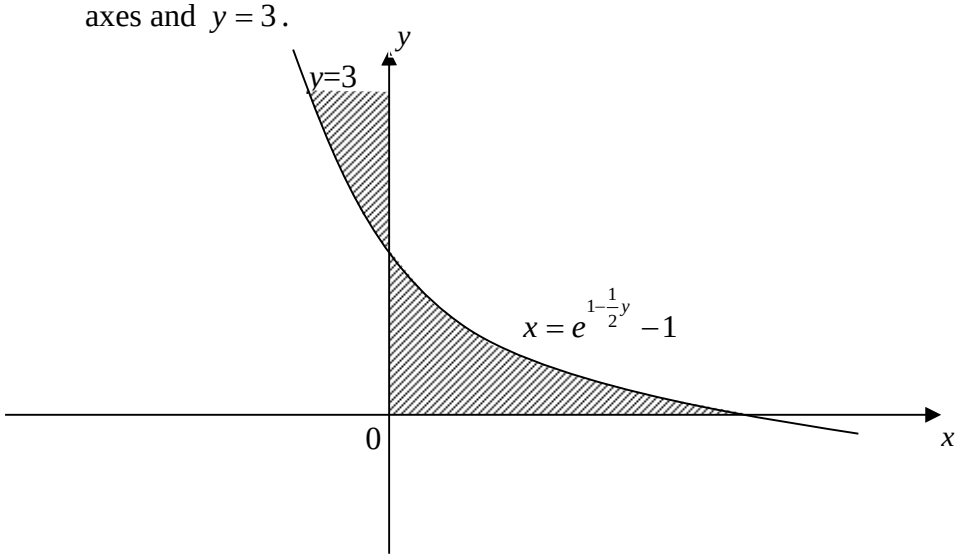
(i) the equation of the normal to the curve at the point $x = \pi$, [2]

(ii) the equation of the curve. [5]

7. The diagram shows part of the curve $x = e^{1-\frac{1}{2}y} - 1$.

(i) Find the coordinates of the point where the curve crosses the *y*-axis. [3]

(ii) Calculate the area of the shaded region bounded by the curve $x = e^{1-\frac{1}{2}y} - 1$, the axes and $y = 3$. [5]



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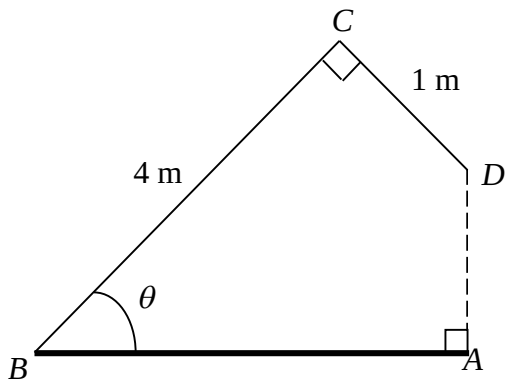


The above diagram shows the side view of a bus stop shelter *BCD* such that *BC* = 4 m, *CD* = 1 m, $\angle BCD = 90°$ and $\angle CBA = \theta$. *AB* is a concrete pavement under the shelter such that *DA* is perpendicular to *AB*.

(i) Show that $AB = 4\cos\theta + \sin\theta$. [2]

(ii) Express *AB* in the form $R\cos(\theta - \alpha)$, where *R* is positive and α is an acute angle. [2]

Hence, find the value of θ for which

(iii) AB^2 is a maximum, [2]

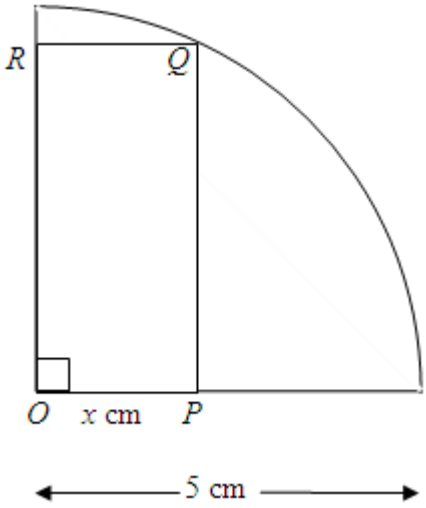
(iv) *AB* = 2 m. [2]

9. **(a)** Solve the equation $\left|x^2 - 6x + 4\right| = 4$. [3]

(b) **(i)** Sketch the graph of $y = \left|x^2 - 6x + 4\right|.$, indicating the y-intercept and the maximum point of the curve. [3]

(ii) Hence, find the range of values of x for which $\left|x^2 - 6x + 4\right| > 4$ [2]

10.



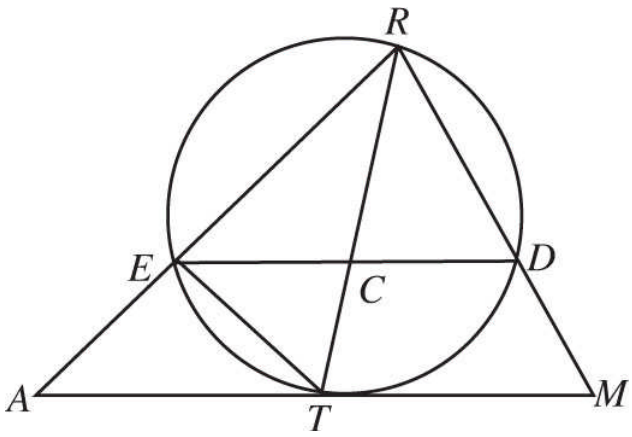
A rectangle *OPQR* is inscribed in a quadrant of a circle of radius 5 cm, two of the sides being along the bounding radii, as shown in the diagram. Given that the length of *OP* is x cm and the area of the rectangle *OPQR* is A cm^2.

(i) Show that $A = x\sqrt{25 - x^2}$. [2]

(ii) Find the value of x for which A is stationary. [3]

(iii) Find the corresponding value of A and determine whether it is a maximum or minimum value. [3]

11.



In the diagram, AM is a tangent to the circle at the point T. Given that REA, RCT and RDM are straight lines. Given further that ED is parallel to AM, $RD = 2DM$ and $RE = \frac{4}{3}RD$.

(i) Show that ΔATE .and ΔART are similar. [2]

(ii) Find the value of $\frac{Area\ of\ \Delta ATE}{Area\ of\ \Delta ART}$. [2]

(ii) Show that $\frac{1}{2}RT^2 = TM \times AT$. [2]

(iii) Show that $3AT^2 = 4RD^2$. [3]

12. A particle moves in a straight line. Its displacement from a fixed point O is given by $S = t^3 - 5t^2 - 8t + 12$, where t seconds is the time from the start of the motion.

(i) Find the value of t when the particle comes instantaneously to rest and the distance the particle has then travelled. [3]

(ii) Find the possible values of t when the particle passes the point O. [3]

(iii) Find the total distance travelled by the particle just before it passes the fixed point O the second time. [2]

(iv) Find the minimum velocity of the particle. [2]

(v) Find the range of values of t for which the velocity of the particle is increasing. [1]

13. In the diagram, *TA* and *TB* are the tangents to the circle $x^2 + y^2 + 4x + 2y - 27 = 0$ at the points *A* and *B*. Given that *C* is the centre of the circle.

(i) Find the centre *C* and the exact value of the radius of the circle. [3]

(ii) Find the length of *TA*. [3]

(iii) Show that $\tan \angle ATC = \frac{2\sqrt{5}}{5}$. Hence find the exact value of $\tan \angle ATB$. [4]

(iv) Find the centre of the circle that passes through *A*, *B* and *T*. [2]