Sec 4 Maths

Exam papers with worked solutions

SET B
PAPER 1
Question

Compiled by

THE MATHS CAFE

1. The gradient of a curve, at the point (x, y) is $3-4x$. Given that the maximum value of the curve is -2 , find the equation of the curve. [4]

2 The variables x and y are related by the equation $y=\ln\left(\sqrt{3+x^2}\right)$.

At the instant when $x = 1$, x is decreasing at the rate of 2 units per second, find the rate of change of y. [4]

3. Find the value of h and of k for which x^2+hx+k is a factor of $2x^3+3x^2+4x-3$.

[4]

4. If $y = x^3 - 3x^2 + 6x + 1$, show that *y* is an increasing function for all real values of *x*. Hence, find the minimum value of the gradient of this function. [5]

5. Differentiate $\dfrac{x^2+1}{2x-3}$ with respect to *x*, and hence evaluate $\int_0^4 \dfrac{x^2-3x}{(2x-3)^2}dx$. [5]

6. A curve has the equation $y = 2\cos\left(2x + \dfrac{\pi}{4}\right)$. Find

(i) an expression for the gradient of the curve [2]

(ii) the *x*-coordinate(s) of the point(s) on the curve at which the gradient of the normal is -0.5 where $0 < x < \dfrac{\pi}{2}$ radians. [4]

7. **(a)** Without using a calculator, evaluate 14^x, given that $2^{3-x} \times 7^{2x-1} = 7^{3x+2}$ [3]

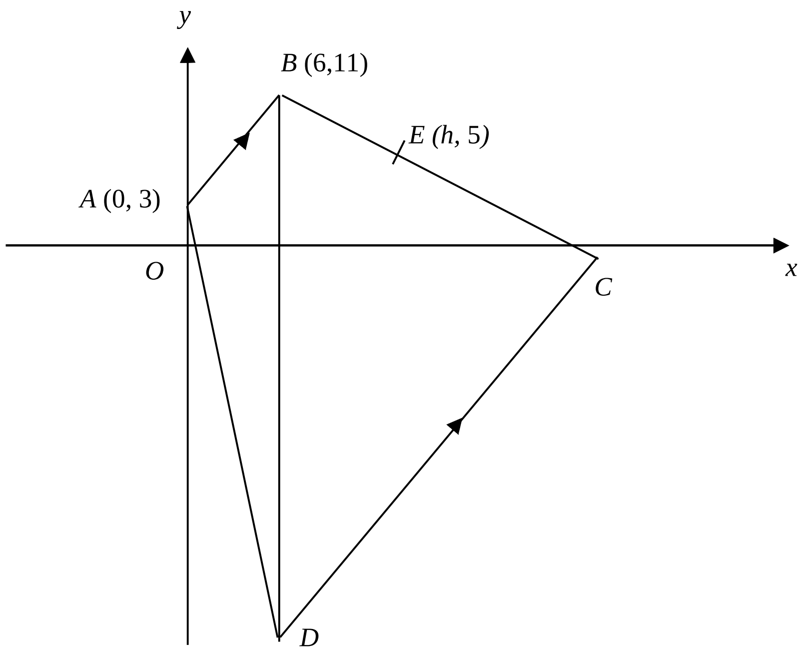
(b) Solve the equation $12\ \log_8(x+2) - \log_{\sqrt{2}}(x+6) = 2$ [4]

8. A quadratic equation is given by $h(x^2+4x) = x - h$, where h is a constant.

(i) Find the range of values of h such that the quadratic equation has real roots. [4]

(ii) Hence, or otherwise, find the range of values of h for which $h(x^2+4x) > x - h$ for all real values of x. [2]

9. Solutions to this question by accurate drawing will not be accepted.



The diagram which is not drawn to scale, shows a trapezium *ABCD* in which *AB* is parallel to *DC*. The coordinates of points *A* and *B* are (0, 3) and (6, 11) respectively. *D* is a point that makes *BD* parallel to the *y*-axis. *E* (h, 5), where h is positive, is a point on *BC* such that $AB = BE$ and $BE : EC = 2:3$.

(i) Show that the value of h is 14. [2]

Find

(ii) the coordinates of *C*, [2]

(iii) the equation of *DC*, [2]

(iv) the coordinates of *D*. [2]

10. A curve is defined by $y = 3\cos\left(\frac{x}{2}\right) - 1$, where $0 \le x \le 4\pi$.

(i) State the period of the curve. [1]

(ii) Write down the minimum value of the curve. [1]

(iii) Determine the *x*-coordinate of the points where the curve crosses the *x* axis [3]

(iv) Sketch the graph of $y = \left|3\cos\left(\frac{x}{2}\right) - 1\right|$ for $0 \le x \le 4\pi$. [3]

(v) Sketch, on the same diagram for $0° \le x \le 4\pi$, the graph of $y = \frac{3x}{2\pi} - 1$.

Hence state the number of roots of the equation $2\pi\left|3\cos\left(\frac{x}{2}\right) - 1\right| = 3x - 2\pi$. [2]

11. **(a)** Find all the angles between 0° and 180° which satisfy the equation
$\cos x - \sin 2x - \cos 3x = 0$ [4]

(b) Solve for $0 < y < 5$, the equation $\sin y \cos y = \cos^2 y - \sin^2 y$. [3]

(c) Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{7}{3}$, show that $2\cot B + 5\cot A = 0$ [3]

12. Answer the whole of this question on a sheet of graph paper.

The population P, in millions, of a country was recorded in the month of January for various years and the results are shown below.

Year	1990	1995	2000	2005	2010
P	122.4	163.2	276.2	278.9	350.0

It is known that $P = 100 + at^n$, where t is the time measured in **years from January 1985** and a and n are constants. An error has been made in recording one of the values of P.

(i) Using the vertical axis for $\lg(P-100)$ and the horizontal axis for $\lg t$, plot $\lg(P-100)$ against $\lg t$ and obtain a straight line graph. [3]

(ii) Use your graph to estimate

(a) the correct reading of P for which an error has been made, [2]

(b) the value of a and of n, [4]

(c) the year in which the population first reached 200 million. [2]