Sec 4 Maths

Exam papers with worked solutions

SET A
PAPER 2
Question

Compiled by

THE MATHS CAFE

Answer **all** the questions

Write in dark blue or black ink on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Write your answers on the writing papers provided.

Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

The use of electronic calculator is expected, where appropriate

You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

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$$\Delta = \frac{1}{2}bc\sin A$$

1. The function f is defined , for $x \geq 0°$, by

$f(x) = 2 - 3\sin 2x$

(i) State the maximum and minimum value of $f(x)$ [2]

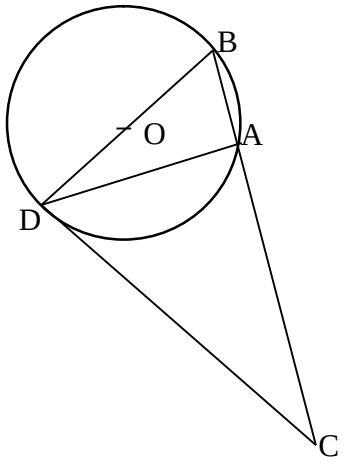
(ii) State the amplitude and period of f. [2]

(iii) Sketch the graph of $f(x) = 2 - 3\sin 2x$ for $0° \leq x \leq 360°$ [2]

2. In the figure below, O is the centre of the circle. DC is a tangent to the circle at the point D and the line BAC is a secant to the circle.

(i) Prove that if BD is a diameter of the circle, then $AD^2 = AB \times AC$ [3]

(ii) Given that $\sin \angle ACD = \frac{1}{2}$, find the exact value of $\frac{AB}{CB}$. [3]



3. (a) Calculate the value of p for which the line $y = 5x - p$ is a tangent to the curve $y = x^2 + 3x + 3$ [3]

(b) Find the smallest integer value of k for which $2x^2 + 5x + k$ is always positive for all real values of x [3]

4. Express $\frac{x+4}{(x+2)(x+1)^2}$ in partial fractions.

Hence evaluate $\int_0^2 \frac{x+4}{(x+2)(x+1)^2} dx$ [7]

5. (a) If α and β are the roots of the equation $2x^2 + hx + 9 = 0$ and $\beta = 2\alpha$, calculate the values of h [3]

(b) Given that α and β are the roots of the equation $3x^2 + 5x - 1 = 0$, form another equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ [5]

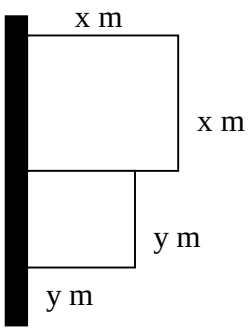
6. A curve is such that $\frac{dy}{dx} = \frac{4x - hx^2}{3}$ where h is a constant. Given that the curve has a turning point at (4, 5)

(i) Show that the value of h is 1. [2]

(ii) Find the range of values of x for which y increases as x increases. [3]

(iii) Find the equation of the curve. [3]

7. A man has 39m of fencing to make two square enclosures using an existing wall as a side of each enclosure. The dimensions of each enclosure are x m and y m (x > y) as shown in the diagram below



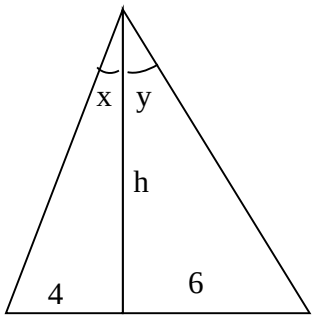
(i) Show that the total area of the two enclosures is $A = x^2 + \frac{(39-3x)^2}{4}$ [2]

(ii) Calculate the value of x at which A has a stationary value. Find this value of A and determine whether it is a maximum or a minimum. [6]

8. (a) Evaluate $\int_0^1 \frac{5+e^{6x}}{e^{4x}} dx$ [4]

(b) Find the equation of the normal to the curve y = $\frac{x-2}{2x+1}$ at the point where the curve crosses the x-axis. [5]

9. (i) The diagram shows a triangle of height h cm and base 10 cm. The angles x and y are such that $x + y = \frac{\pi}{4}$. By using the expansion of $\tan(x + y)$, or otherwise, find the value of h. [4]



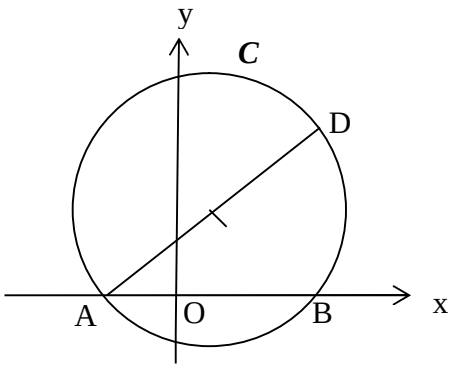
(ii) Prove that $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$. Hence solve the equation

$\cot\theta + \frac{\sin\theta}{1+\cos\theta} = 2$ for $0 < \theta < 2\pi$ [5]

10. (a) Given that $(x - 1)$ and $(x + 2)$ are factors of the expression $3x^3 + hx^2 - kx - 10$, find the value of h and of k. Hence, find the remainder when the expression is divided by $(2x - 1)$ [5]

(b) Factorise the expression $6x^3 - 17x^2 + 11x - 2$ completely. Hence, solve the equation $6(k + 2)^3 - 17(k + 2)^2 + 11k + 20 = 0$ [5]

11. The equation of the circle, *C*, as shown in the diagram below , is
$x^2 + y^2 - 6x - 10y - 135 = 0$



(i) Find the coordinates of the centre of ***C*** and find the radius of ***C*** [3]

(ii) Given that the circle cuts the x-axis at the points A and B. Find the length of the line segment AB. [3]

Given that D is a point on the circle such that the line segment AD is the diameter of the circle.

(iii) Find the coordinates of D. [1]

(iv) Find the equation of AD. [2]

(v) Find the equation of the circle which is a reflection of ***C*** in the y-axis [2]

12.(a) Solve the simultaneous equations

$5^x (25^y) = 0.2$

$\log_2 (y - x) - 2 = \log_2 (x + 4)$ [5]

(b) (i) Find the values of m for which $\begin{pmatrix} 4 & -m \\ m & -1 \end{pmatrix}$ is a singular matrix [2]

(ii) Given that $A = \begin{pmatrix} 8 & 5 \\ 4 & 3 \end{pmatrix}$. Find A^{-1}

Hence solve the simultaneous equations

$8x + 5y - 14 = 0$

$3y + 4x = 6$ [5]

---------- END OF PAPER ----------