Sec 4 Maths

Exam papers with worked solutions

SET A
PAPER 1
Question

Compiled by

THE MATHS CAFE

Answer **all** the questions

Write in dark blue or black ink on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Write your answers on the writing papers provided.

Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

The use of electronic calculator is expected, where appropriate

You are reminded of the need for clear presentation in your answers.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1. A particle moves in a straight line from a fixed point O such that t seconds after leaving O, its velocity , v m/s is given by $v = 10(1 - e^{-2t})$. Calculate

   (i) the acceleration of the particle when t = 1. [2]

   (ii) the displacement of the particle from O when t = 1. [3]

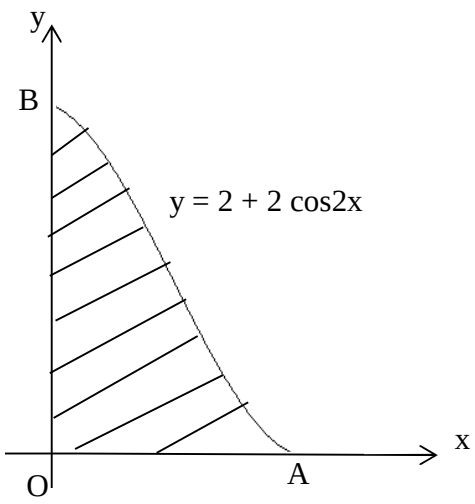
2. (i) Find the coordinates of all the points at which the graph of $y = |3 - \frac{x}{2}| - 1$ meets the coordinates axes. [3]

   (ii) Hence sketch the graph of $y = |3 - \frac{x}{2}| - 1$ for $-2 \le x \le 9$ and state the range of values of y. [3]

3. The diagram shows part of the curve $y = 2 + 2\cos 2x$, meeting the x-axis at A and y- axis at B

(i) Show that the x-coordinate of A is $\frac{\pi}{2}$ and find the y-coordinate of B. [3]

(ii) Hence find the area of the shaded region. [3]

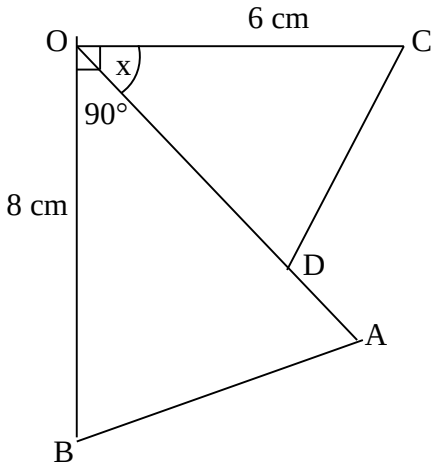


4. (a) Find the coefficient of x^2 in the expansion of $(2x - \frac{1}{3x^2})^8$. [3]

(b) In the Binomial expansion of $(1 + ax)^n$, the first three terms are $1 + 3x + 4x^2 + \ldots$. Calculate the value of n and of a. [4]

5. The diagram shows two isosceles triangles AOB and COD. Given that

OA = OB = 8 cm, OC = OD = 6 cm, $\angle$ COB= 90° and $\angle$ COD = x°.



The sum of the areas of Δ AOB and Δ COD is A cm^2.

(a) Show that A = 18 sin x + 32 cos x [2]

(b) Find the value of R and α for which A = R sin (x + α), where R > 0 and $0° < \alpha < 90°$ [2]

Given that x can vary, find

(c) the maximum value of A and the corresponding value of x. [2]

(d) the value of x for which the area of Δ AOB is twice the area of Δ COD. [2]

6. Solve the following equations

(i) $\log_3 (3^{x+1} - 8) = x$. [4]

(ii) $\log_4 x + 2 \log_x 4 = 3$. [4]

7. Given that $y = (x - 1)\sqrt{2x+7}$.

(i) Show that $\frac{dy}{dx} = \frac{3x+6}{\sqrt{2x+7}}$. [3]

(ii) Hence evaluate $\int_1^9 \frac{x+2}{\sqrt{2x+7}}\, dx$. [3]

(iii) Find the rate of change of x when x = 1, given that y is changing at a constant rate of 6 units per second. [3]

8. The table shows experimental values of two variables x and y.

x	1.6	1.8	2	3.4	5
y	0.65	0.82	1	2.5	4.7

It is known that x and y are related by an equation of the form $\frac{p}{y} - \frac{q}{x^2} = 1$, where p and q are constants.

Plot the straight line graph of $\frac{1}{y}$ against $\frac{1}{x^2}$ for the given data and use your graph to estimate

(i) the value of p and of q,

(ii) the value of x when $\frac{2}{y} = 3$. [9]

9. (a) Given that sin (x + b) + cos (x + b) = cos x where b is the acute angle such that tan b = $\frac{3}{4}$, find the value of tan x [4]

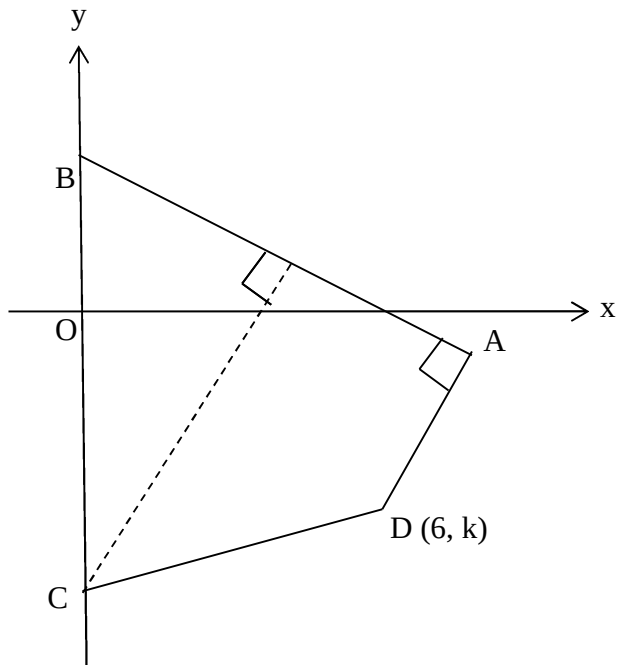
(b) If cos 2θ = – $\frac{4}{5}$ and 90° ≤ θ ≤ 135°, calculate, without using the calculator, the value of

(i) cos θ

(ii) sin 2θ

(iii) sin 4θ. [6]

10. Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a quadrilateral ABCD. The equation of line AB is $2y + x - 6 = 0$, and the y- coordinate of A is -1. The point B lies on the y-axis. The perpendicular bisector of AB meets the y-axis at the point C. The coordinates of D is (6, k) and AD is perpendicular to AB.

(i) Find the coordinates of the point A. [1]

(ii) Find the value of k. [2]

(iii) Find the coordinates of the points B and C. [3]

(iv) Find the equation of the line parallel to AB, passing through C. [2]

(v) Calculate the area of the triangle ABC, and hence find the distance of C from the line AB. [4]

---------- END OF PAPER ----------