

# **H2 MATHS**

**Exam papers with worked solutions**

## **SET D PAPER 2**

Compiled by

**THE MATHS CAFE**

**Section A: Pure Mathematics [40 marks]**

**1** A sequence of positive real numbers  $x_1, x_2, x_3, \dots$  satisfies the recurrence relation

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{3}{x_n} \right)$$

for  $n \geq 1$ .

As  $n \rightarrow \infty$ ,  $x_n \rightarrow l$ .

- (i) Find the exact value of  $l$ . [2]
- (ii) Show that if  $x_n > l$ , then  $x_n > x_{n+1} > l$ . [4]

- 2** A curve is given parametrically by the equations  $x = 2t - 1$ ,  $y = \frac{1}{2t + 1}$ , where  $t \in \mathbb{R}$ .
- (a) Show that there is no tangent to the curve parallel to the chord joining the points where  $t = -1$  and  $t = \frac{5}{8}$ . [3]
- (b) Find the coordinates of the point at which the normal to the curve at  $t = \frac{1}{2}$  meets the curve again. [4]

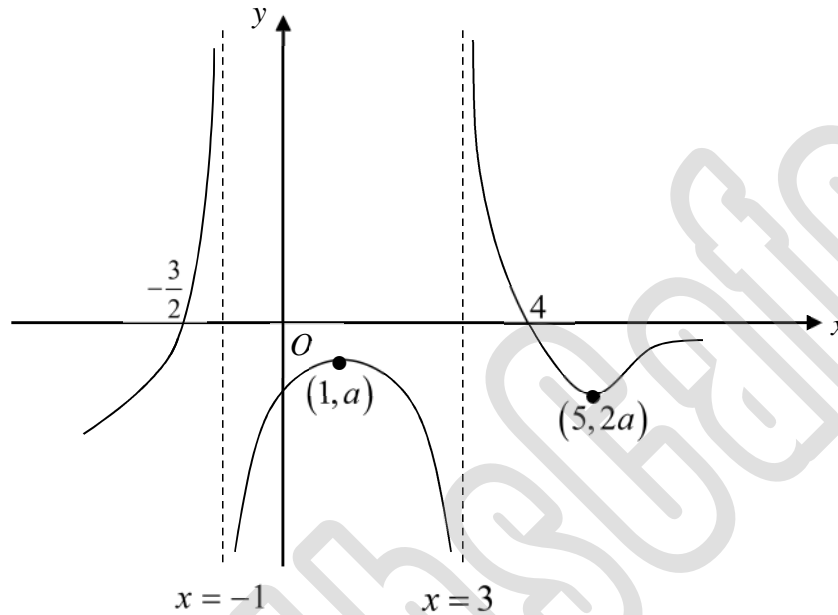
- 3** Using the substitution  $z = ye^x$ , find the general solution of the differential equation  $\frac{dy}{dx} + 3x + y = 0$ , giving our answer in the form  $y = f(x)$ . [5]

Sketch, on the same diagram, two solution curves such that one is linear and the other has a turning point. [3]

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- 4** (i) Solve the equation  $iz^6 = -\frac{64}{\sqrt{2}}(1+i)$ , giving the roots in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . [4]
- (ii) Express the complex number  $w = z^6 + \frac{1}{(z^*)^6}$  in trigonometric form and find the set of values of the positive integer  $n$  for which  $w^n$  is real. [4]
- (iii) Hence, show that for these values of  $n$ , the values of  $w^n$  is either  $\left(\frac{4097}{64}\right)^n$  or  $-\left(\frac{4097}{64}\right)^n$ . [1]

- 5 (a) Describe a sequence of transformations which transforms the graph of  $x^2 + y^2 = 1$  to that of  $x^2 + (2y+2)^2 = 4$ . [3]
- (b) The diagram below shows the graph of  $y = f(x)$  where  $a < 0$ .



Sketch, on separate clearly labelled diagrams, the graphs of

- (i)  $y = -\sqrt{f(x)}$ , [3]
- (ii)  $y = \frac{2}{f(x)}$ . [4]

Your sketch should include the axial intercepts, coordinates of turning points, and equations of asymptotes.

**Section B: Statistics [60 marks]**

- 6** A group of students from the Mass Communication faculty in Valley University came down to Lee Hwa Junior College to conduct a survey to find out the impact of social media on the results of the General Elections. A sample of 50 students was selected in the canteen during the students' lunch break, with 20 students from JC1 and 30 students from JC2.

State the sampling method used. [1]

Describe how systematic sampling could be used instead to gather data from 50 students, assuming that the college has a total population of 2000 students. State one advantage of systematic sampling over the previous method. [3]

- 7** A factory produces packets of peanuts. The mass of peanuts in a packet has mean 605g and standard deviation 6g. A sample of sixty packets is chosen. Find the probability that the mean mass of peanuts in a packet from this sample is between 600g and 606g. State the assumptions that you have made. [4]

- 8** (a) Find the number of ways of arranging the word PRELIMINARY if the first and last letter is a vowel. [4]
- (b) A group of 10 students comprising of 3 males and 7 females attended their graduation dinner. Find the number of ways of seating them at a round table with 10 chairs of different colors and no two male students are seated next to each other. [3]



- 9 *The RVmart Show* is an upcoming weekly TV variety show sponsored by the RV hypermart. The variety show comprises a series of games from which contestants stand to win cash prizes. In one of the game segments named *Box of Fortune*, a contestant randomly selects four balls without replacement from a box. There are 4 red, 5 yellow, 2 blue and 1 black balls in the box. The balls are indistinguishable except for their colour.

Except for the black ball, each ball is assigned a cash value as follows:

Red: \$1000                      Yellow: \$500                      Blue: \$0

If no black ball is selected, the total cash values of the four balls will be the total cash winnings of the contestant. However, in the unfortunate event that the black ball is selected, the total cash winnings of the contestant will be half of the total cash values of the remaining three balls.

Find the probability that a contestant

- (i) wins the maximum possible amount of cash, [2]  
(ii) has a total cash winnings of \$1000, [3]  
(iii) selects exactly 2 blue balls given that his total cash winnings is \$1000.

[2]

- 10** Each night, an astrologer captures a large portion of the sky with a videoing device and records the number of meteors that appear. State a condition under which a Poisson distribution is a suitable model. [1]

The mean number of meteors seen per hour during the night is  $\lambda$ . Assuming a Poisson distribution, find the least integer value of  $\lambda$  if there is a probability of more than 0.95 that at least 2 meteors seen during a 2-hour period at night. [4]

Suppose for each night, the video recording is done from 10 p.m. till 4 a.m. the following morning and that the value of  $\lambda$  is 3. Using a suitable approximation, find the probability that the video recording captures exactly 20 meteors during a night. [3]

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- 11** The amount of radioactive material  $x$  (in grams) left after time  $t$  (in seconds) is shown in the following set of data below.

$t$ (seconds)	5	10	15	20	25	30
$x$ (grams)	1.321	0.507	0.251	0.196	0.053	0.046

- (i) Draw a scatter diagram for these values and comment on the relationship between  $t$  and  $x$ . [2]

It is thought that  $x$  and  $t$  can be modelled by one of the formulae

$$\ln x = a + bt \quad \text{or} \quad \ln x = c + dt^2$$

where  $a$ ,  $b$ ,  $c$  and  $d$  are constants.

- (ii) Find the value of the product moment correlation coefficient between
- (a)  $t$  and  $\ln x$ ,
- (b)  $t^2$  and  $\ln x$ . [2]
- (iii) Use your answer to part (ii) to explain which of  $\ln x = a + bt$  or  $\ln x = c + dt^2$  is the better model. [1]
- (iv) By using a suitable regression line, estimate the value of  $t$  for which  $x = 0.123$ . Explain your choice of regression line. [3]

- 12** A typical lesson at Lim Tuition Centre consists of lecture and tutorial. The durations of lecture and tutorial may be assumed to have independent normal distributions with the following means and standard deviations.

	Mean (hours)	Standard deviation (hours)
<b>Lecture duration</b>	0.8	$\sigma$
<b>Tutorial duration</b>	1.1	0.195

A lesson is said to be overly long if its total duration exceeds 2.3 hours.

- (i) Given that 90% of the lessons have lecture duration of at most 1 hour, find the value of  $\sigma$ . [3]
- (ii) Find the probability that in an overly long lesson the duration of the lesson is at most 2.5 hours. [3]
- (iii) One hundred lessons are sampled randomly from the centre. Find the probability that less than ten lessons are overly long. [2]
- (iv) Tan Tuition Centre also conducts its lessons consisting of lecture and tutorial. The total duration of a lesson has a normal distribution with mean 2.2 hours and standard deviation 0.4 hour. The Lim and Tan centres charge their pupils at rates of \$20 and \$19 per hour respectively. Assuming cost is the only factor, determine which centre should a pupil sign up with. [3]

- 13** The standby-time,  $x$  hours, of each fully charged battery in a random sample of 12 imyPhones is measured. The total standby time is 1236 hours and the sample has a standard deviation of 8.5 hours. Find the unbiased estimates of the population mean and variance. [2]

Stating a necessary assumption, carry out a test at the 10% level of significance whether the mean standby-time of a fully charged imyPhone battery is at most 100 hours. Briefly explain the meaning of the  $p$ -value obtained in context of the question. [6]

In another test, using the same data and also at 10% level of significance, the hypotheses are as follows:

Null hypothesis: the population mean standby time of imyPhone batteries is equal to  $\mu_0$

Alternative hypothesis: the population mean standby time of imyPhone batteries is not equal to  $\mu_0$

Taking the population standard deviation to be 8 hours, find the set of possible values of  $\mu_0$  given that the null hypothesis is rejected in favour of the alternative hypothesis. [3]