# H2 MATHS

**Exam papers with worked solutions**

# SET A

# PAPER 2

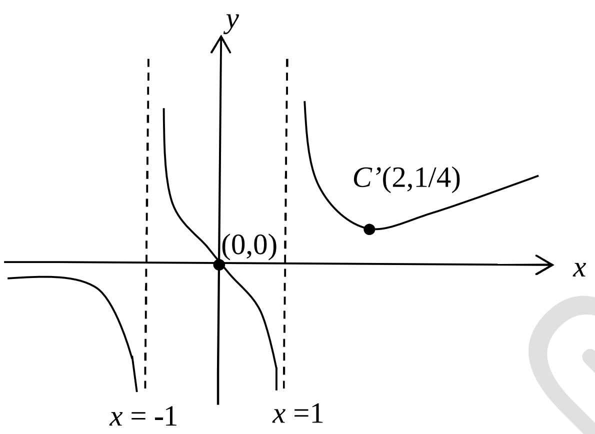
## ANSWER

**Compiled by**

# THE MATHS CAFÉ

Section A: Pure Maths

1(a)




(b)

Let $h(x)=\dfrac{x-1}{x+1}$. Thus the equation of the resulting graph is y = h($x$). To obtain the original graph we need to work backwards and reverse each transformation, starting with the last.

| | | |
|---|---|---|
| C': | Stretch with scale factor ½ parallel to $x$-axis<br>$y=h(x)\rightarrow y=h(2x)$<br>(Replace $x$ with 2$x$) | $y=\dfrac{2x-1}{2x+1}$ |
| B': | Reflection in the $y$-axis<br>$y=h(2x)\rightarrow y=h(2(-x))$<br>(Replace $x$ with -$x$) | $y=\dfrac{-2x-1}{-2x+1}$ |
| A': | Translation of 4 units in the negative x-direction.<br>$y=h(-2x)$<br>$\rightarrow y=h\left(-2(x+4)\right)$<br>(Replace $x$ with $x$+4) | $y=\dfrac{-2(x+4)-1}{-2(x+4)+1}$<br>$y=\dfrac{-2x-9}{-2x-7}$<br>$y=\dfrac{2x+9}{2x+7}$ |

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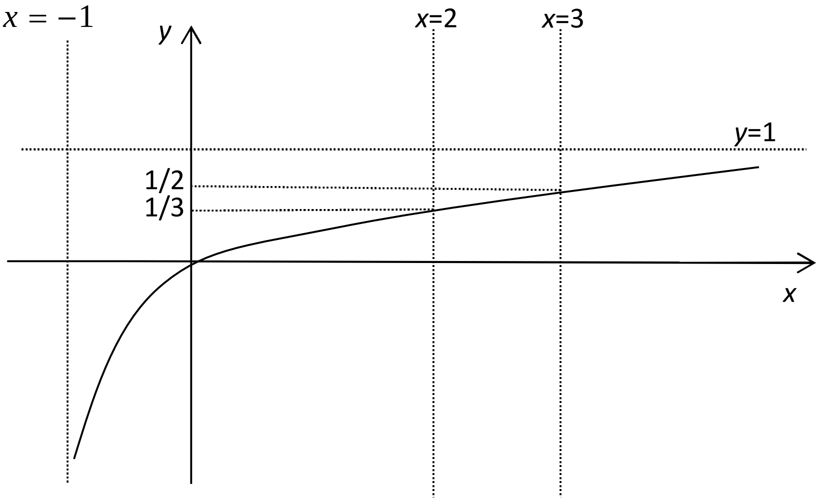
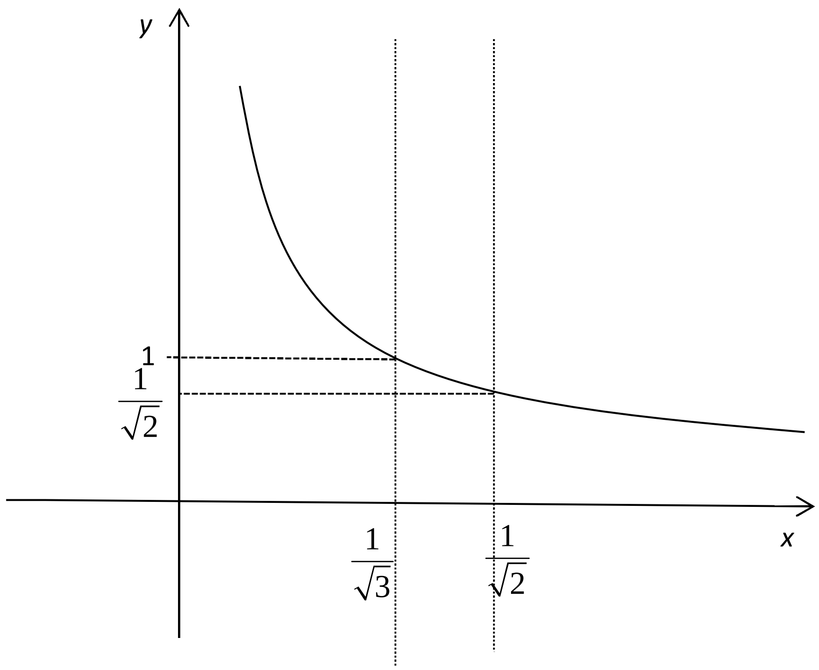
| 2a | <br>The intersection points are (2,1/3) and (3,1/2).<br>Volume required$=\pi\left(3^2\right)\left(\frac{1}{2}\right)-\pi\int_{1/3}^{1/2}x^2dy-\pi\left(2^2\right)\left(\frac{1}{3}\right)$<br>$=\frac{9\pi}{2}-\pi\int_{1/3}^{1/2}\left(\frac{1+y}{1-y}\right)^2dy-\frac{4\pi}{3}$<br>$=\frac{19\pi}{6}-\pi\left(1.0159\right)$<br>$=6.76\text{ units}^3$ |
| 2b |  |

When $x=\frac{1}{\sqrt{3}}, t=-\sqrt{3}, y=1$

When $x=\frac{1}{\sqrt{2}}, t=-\sqrt{2}, y=\frac{1}{\sqrt{2}}$

$$\frac{dx}{dt}=\frac{1}{t^2}$$

$$\int_{\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{2}}} y \, \mathrm{d}x$$

$$=\int_{-\sqrt{3}}^{-\sqrt{2}} \frac{1}{\sqrt{4-t^2}} \frac{\mathrm{d}x}{\mathrm{d}t} \mathrm{d}t$$

$$=\int_{-\sqrt{3}}^{-\sqrt{2}} \frac{1}{\sqrt{4-t^2}} \frac{1}{t^2} \mathrm{d}t$$

$a=-\sqrt{3}$ b= $-\sqrt{2}$

Using t = 2sin $\theta$

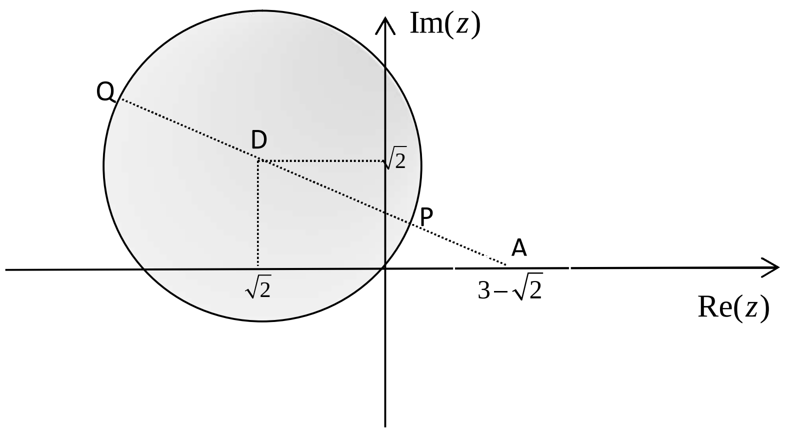
When $t=-\sqrt{3}, \theta=-\frac{\pi}{3}$

When $t=-\sqrt{2}, \theta=-\frac{\pi}{4}$

$$\frac{dt}{d\theta}=2\cos\theta$$

$$\int_{-\sqrt{3}}^{-\sqrt{2}} \frac{1}{\sqrt{4-t^2}} \frac{1}{t^2} \mathrm{d}t$$

$$=\int_{-\frac{\pi}{3}}^{-\frac{\pi}{4}} \frac{1}{\sqrt{4-4\sin^2\theta}} \frac{1}{2^2\sin^2\theta} 2\cos\theta \, \mathrm{d}\theta$$

| | |
|---|---|
| | $=\frac{1}{4}\int_{-\frac{\pi}{3}}^{-\frac{\pi}{4}} \operatorname{cosec}^2\theta \, d\theta$<br><br>$=-\frac{1}{4}[\cot\theta]_{-\frac{\pi}{3}}^{-\frac{\pi}{4}}$<br><br>$=-\frac{1}{4}\left(-1+\frac{1}{\sqrt{3}}\right)=\frac{1}{4}\left(1-\frac{1}{\sqrt{3}}\right)$ unit$^2$<br><br>(=0.382) |
| 3(a) | Since $z=1+\sqrt{2}i$ is a root, $z=1-\sqrt{2}i$ is another root. Thus<br><br>$z^3+az^2+bz-6=\left(z-1-\sqrt{2}i\right)\left(z-1+\sqrt{2}i\right)(z-c)$<br>$=\left(z^2-2z+3\right)(z-c)$<br><br>By comparing coefficients, we have *c* = 2.<br><br>Thus we have<br><br>$z^3+az^2+bz-6=\left(z^2-2z+3\right)(z-2)$<br>$=z^3-4z^2+7z-6$<br><br>Thus a=$-4$ and b=7. |
| 3(b) |  |
| (i) | Minimum $\left\|z-3+\sqrt{2}\right\|=AP=AD-DP=\sqrt{11}-2$<br><br>Maximum $\left\|z-3+\sqrt{2}\right\|=AQ=AD+DQ=\sqrt{11}+2$ |

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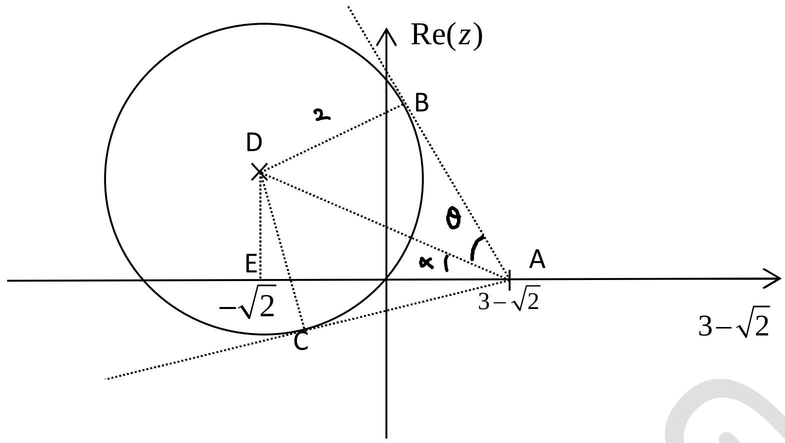
| (ii) | <br><br>$\text{DA} = \sqrt{11}$<br>$\sin\theta = \frac{2}{\sqrt{11}} \Rightarrow \theta = 0.647 \text{ rad}$<br>$\tan\alpha = \frac{\sqrt{2}}{3} \Rightarrow \alpha = 0.441 \text{ rad}$<br>When $\text{P}(\equiv z)$ is at B, $\arg(z) = \pi - \theta - \alpha = 2.05 \text{ rad}$<br>When $\text{P}(\equiv z)$ is at C, $\arg(z) = -[\pi - (\theta - \alpha)] = -2.94 \text{ rad}$<br>Thus, $2.05 \le \arg(z) \le \pi$ or $-\pi < \arg(z) \le -2.94$ |
| | |
| 4a) | $\frac{x^2-3}{x^2+2x-1} = \frac{x^2-2x+1}{x^2-3}$<br>$\Rightarrow (x^2-3)^2 = (x^2+(2x-1))(x^2-(2x-1))$<br>$\Rightarrow x^4 - 6x^2 + 9 = (x^4 - (2x-1)^2)$<br>$\Rightarrow -2x^2 - 4x + 10 = 0$<br>$\Rightarrow x^2 + 2x - 5 = 0$<br>$\Rightarrow x = \frac{-2 \pm \sqrt{4 - 4(1)(-5)}}{2}$<br>$\therefore \quad x = -1 - \sqrt{6} \quad \text{or} \quad -1 + \sqrt{6}$<br>If $x = -1 + \sqrt{6}$, $r = 2.23$ (rejected as the sequence converges) |

If $x = -1-\sqrt{6},\ r = -0.225$

The common ratio, $r =$ - 0.225 (3 sig figs).

b)

| Sapling No | Distance walked by farmer |
|---|---|
| 1 | 0 |
| 2 | 2(1) |
| 3 | 2(2) |
| 4 | 2(3) |
| … | … |
| 200 | 2(199) |
| 201 | 200 |

Distance covered followed an A.P with common difference 2.

Therefore, total distance = (2+4+…+398)+200

$$= 2(1+2+\ldots+199)+200$$

$$= 2\left(\frac{199}{2}(1+199)\right)+200$$

= 40000 m. (shown)

| Sapling No | Distance walked by farmer |
|---|---|
| 1,2 | 2(1) |
| 3,4 | 2(3) |
| 5,6 | 2(5) |
| … | … |
| 199,200 | 2(199) |
| 201 | 200 |

Distance covered followed an A.P with common difference 4.

Therefore, total distance = (2+6+10+…+199) + 200

$$= 2(1+3+5+\ldots+199)+200$$

$$= 2\left(\frac{100}{2}(1+199)\right)+200$$

= 20200 m.

Hence distance saved by the famer by using the $2^{nd}$ method is (40000 – 20200)m = 19800 m.

## Section B: Statistics

5(i) Quota sampling.

(ii) Possible answers :

(1)Information can be collected quickly as a sampling frame is not required.

(2)Cost is low.

(3)Ensures selection of adequate numbers of participants with appropriate

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| | characteristics. |
| (iii) | Stratified sampling method. This gives a more representative sample.<br><br>Or simple random sampling method. The data collected is free from bias as every member of the population has an equal chance of being selected.<br><br>Or systematic sampling method. Sample is more evenly spread over the population of working adults as the sample is taken throughout the ordered population. |
| 6 | (i) No of ways= $9! = 362,880$<br><br>(ii) Arrange the 6 non-Tan diners first. No of ways to do that=6!<br><br>We can slot the 3 Tans in between the diners in $^7P_3$ ways.<br><br>Total no of ways= 6! x $^7P_3$ = 151200<br><br>(iii)<br><br>No of ways to choose diners for the four-seater= $^9C_4$ =126 ways<br><br>No of ways to seat the diners= (4-1)!= 3!<br><br>No of ways to choose diners for five-seater = $^5C_5$ =1<br><br>No of ways to seat the diners= (5-1)!=4!<br><br>Total no of ways = 18144<br><br>(iv)<br><br>Case 1: David is seated at four-seater<br><br>No of ways to seat David at four-seater= $^6C_1 \times (2-1)! \times (2!) = 12$<br><br>No of ways to seat the rest at five-seater= (5-1)!=4!=24<br><br>Total no of ways (for Case 1)= 12 x 24 = 288<br><br>Case 2: David is seated at five-seater<br><br>No of ways to seat David at five-seater= $^6C_2 \times (3-1)! \times (2!) = 60$<br><br>No of ways to seat the rest at four-seater = (4-1)!=3!=6<br><br>Total no of ways (For Case 2)= 6 x 60 = 360 |

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| | Total no of ways= 288+360= 648 |
| 7 (i) | $s^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n-1} = \frac{180}{59} = 3.05$ (3s.f) |
| (ii) | Let $X$ be the r.v. "weight of a randomly chosen packet of cotton candy".<br><br>Let $\mu$ be the population mean of X.<br>Test $H_0 : \mu = \mu_0$<br>Against $H_1 : \mu \neq \mu_0$<br>$\sigma$ unknown, $n$ = 60. 2-tailed z-test is used at at 5% significance level,<br><br>Since $n > 50$<br>Under $H_0$, using Central Limit Theorem,<br>$Z = \frac{\bar{X} - \mu_0}{\sqrt{s^2 / n}} \sim N(0,1)$ approximately<br><br>Since $\text{H}_0$ is NOT rejected at 5% level of significance,<br><br>$-1.95996 < \frac{35-\mu_0}{\sqrt{3.0508/60}} \leq 1.95996$<br>$-1.95996\sqrt{\frac{3.0508}{60}} - 35 < -\mu_0 < 1.95996\sqrt{\frac{3.0508}{60}} - 35$<br><br>$34.6 < \mu_0 < 35.4$ (3 s.f) |
| (iii) | Let y represent the weight of the 5 packets.<br>$\sum\left(y-35\right)^2 = 8.38$<br>$\bar{y} = 35$<br>Combining into a sample of 65 packets,<br>$\bar{x} = 35$ and $\sum\left(x-35\right)^2 = 180 + 8.38 = 188.38$<br>$s^2 = \frac{188.38}{64} = 2.9434375$<br>Under $H_0$<br>$H_0 : \mu = 35.7$<br>$H_1 : \mu \neq 35.7$<br><br>Under $H_0$, $\bar{X} \sim N\left(35.7, \frac{2.9434375}{65}\right)$ by CLT<br><br>Using GC |

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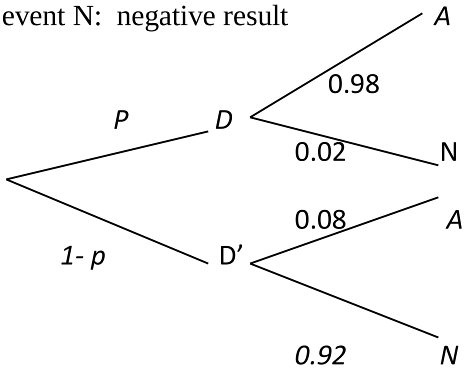
| | $p-\text{value}=0.00100<0.02$<br>Since $p$-value < 0.02, reject $H_0$<br>There is sufficient evidence at 2% level of significance level to conclude that the mean weight of cotton candies is not 35.7g. |
| 8 | Let event D: person infected with disease<br>event A: positive result<br>event N: negative result<br><br> |
| (i) | P($D\|A$) = $\dfrac{P(D\cap A)}{P(A)} = \dfrac{0.98p}{0.98p+0.08(1-p)}$<br>$= \dfrac{0.98p}{0.9p+0.08}$ (Ans) |
| (ii) | P(*wrong conclusion*) = P(*D,N*) + P(*D', A*)<br>= 0.02*p* + 0.08(1-*p*)<br>= 0.08- 0.06p (Ans) |
| (iii) | P(individual has the disease\| result of 2nd test is positive)<br>$= \dfrac{P(D,A,A)}{P(D,A,A)+P(D',A,A)}$<br>$= \dfrac{0.98^2 p}{0.98^2 p+0.08^2 p}$. (Ans)<br>$= \dfrac{0.9604p}{0.0064+0.954p}$ |
| (iv) | Let X be the r.v. "no of people who will take the test twice".<br>$X \sim B(12000, 0.08)$<br>Expected no of people who has to take the test twice= np = 960 |
| 9 (i) | The occurrences of failed pixels are independent of one another.<br>The average number of failed pixels for each TV is constant. |
| (ii) | Let $X$ be r.v. "no. of failed pixels in 1 LCD screen." |

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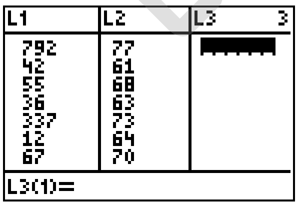
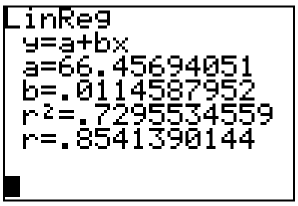
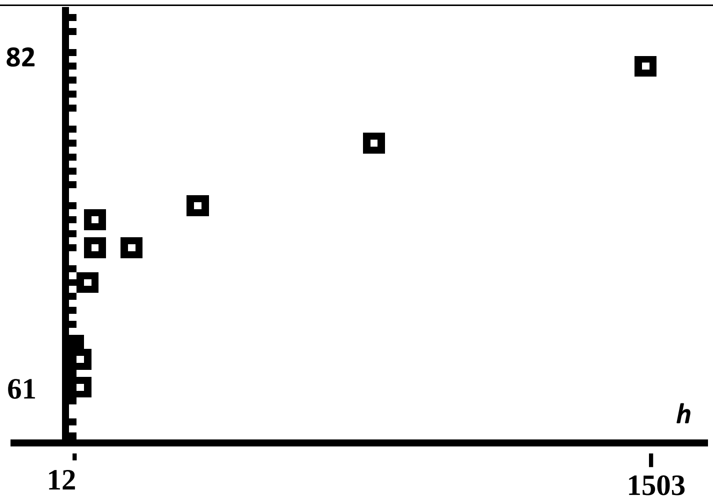
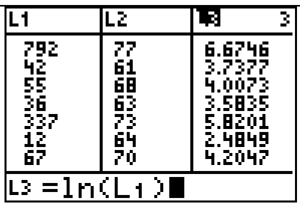
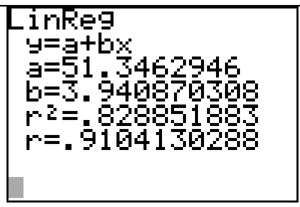
| | $X \sim Po(0.2)$<br>$X_1+...+X_{10} \sim Po(2)$<br>$P(3 < X_1+...+X_{10} \le 6)$<br>$= P(X_1+...+X_{10} \le 6) - P(X_1+...+X_{10} \le 3)$<br>$= 0.138$ |
| (iii) | $P$(imperfect LCD screen) = $P(X \ge 1)$<br>$= 1 - P(X = 0) = 0.18127$ |
| | Let $Y$ be r.v. “no. of imperfect screens in a batch of 50”.<br>$Y \sim B(50, 0.18127)$<br>$n$ large, $np = 9.0635 > 5, nq = 40.9365 > 5$<br>$Y \sim N(9.0635, 7.4206)$ approximately<br>$P(Y = 10)$<br>$\xrightarrow{c.c} P(9.5 < Y < 10.5)$<br>$= 0.137$ |
| (iv) | $\overline{Y} \sim N(9.0635, \frac{7.4206}{52})$ approx by CLT (since n > 50 is large)<br>$P(\overline{Y} > 8) = 0.998$ |
| 10 (i) | Let $C$ and $B$ be random variables “weekly earnings (in thousands of dollars) for cars and buses respectively”.<br>$C \sim N(120.3, 10.4^2)$ and $B \sim N(69.2, 12.5^2)$<br>$C - B \sim N(51.1, 10.4^2 + 12.5^2)$<br>$C - B \sim N(51.1, 264.41)$<br>$P(\lvert C - B \rvert \le 60) = P(-60 \le C - B \le 60) = 0.708$ |
| (ii) | Let $L$ be random variable “weekly earnings (in thousands of dollars) for lorries”.<br>$L \sim N(64.5, 9.5^2)$<br>$L_1 + L_2 + ... + L_5 \sim N(5(64.5), 5(9.5^2)) = N(322.5, 451.25)$<br>$P(L_1 + L_2 + ... + L_5 > 345) = 0.145$<br>The weekly earnings for lorries are independent of one another in the |

| | |
|---|---|
| | 5-week period. |
| (iii) | Let $T$ be random variable "total amount for repairs (in thousands of dollars)".<br><br>$T=\frac{x}{100}C+0.08B+0.15L$<br>$\sim N(120.3\left(\frac{x}{100}\right)+15.211, 108.16\left(\frac{x}{100}\right)^2+3.030625)$<br><br>$\mathrm{P}(T\geq 25)=0.097$<br>$P(T<25)=0.903$<br>$P(Z<\frac{25-E(T)}{\sqrt{Var(T)}})=0.903$<br>$\frac{9.789-120.3\frac{x}{100}}{\sqrt{108.16\left(\frac{x}{100}\right)^2+3.030625}}=1.2988$<br>From GC, $x=6.14$ |
| (iv) | If a normal distribution is used to model the distribution of weekly takings, we would expect $\mathrm{P}(2-3(0.8)<X<2+3(0.8))=0.998$ i.e.<br><br>$\mathrm{P}(-0.4<X<4.4)=0.998$. However this is not reasonable as there is a significant range of weekly takings that are negative amounts. Hence the weekly takings should not follow a normal distribution.<br><br>Alternatively,<br><br>If $X\sim N(2000, 800^2)$, $P(X<0)=0.00621$ (which is too large) |
| 11 i) |   |

<table>
<tr><td></td><td>$r = 0.854$<br><br>Since $r = 0.854$, there is a strong linear relationship between healthcare expenditure and life expectancy.</td></tr>
<tr><td>ii)</td><td>
</td></tr>
<tr><td>iii)</td><td>The scatter diagram shows a logarithm relationship between $l$ and $h$.</td></tr>
<tr><td>iv)</td><td>There is a limit to how money can improve one's life expectancy. If the $l$ and $h$ follows a linear relationship, it would then be possible to live very long with enough money.<br><br>Or any other logical explanation</td></tr>
<tr><td>v)</td><td>



<br>$a = 51.3, b = 3.94$</td></tr>
<tr><td>vi)</td><td>$l = 51.3462946 + 3.940870308\ln(h)$</td></tr>
</table>

| | |
|---|---|
| | When $h=500$, $l=51.3462946+3.940870308\ln\left(500\right)=75$<br><br>Since $r=0.910$ and $h=500$ is in range of $12\le h\le 1503$, interpolation is done. The prediction is valid. |
| vii) | His statement is not valid since linear regression does not claim casualty.<br><br>Or<br><br>The conditions in Indonesia are different from other countries. |