

# **H2 MATHS**

**Exam papers with worked solutions**

# **SET A**

# **PAPER 2**

Compiled by

# **THE MATHS CAFE**

## READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks is 100.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically state otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematic steps using mathematical notations and not calculator commands.

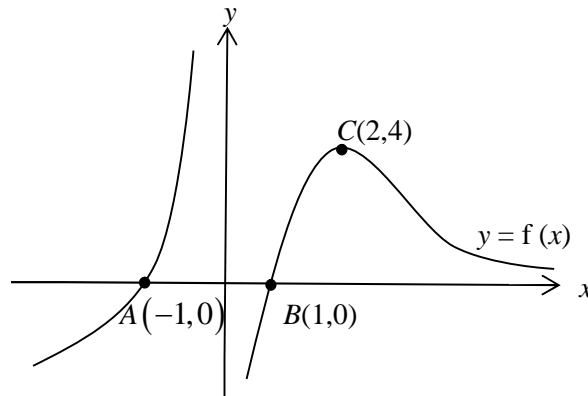
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

**Section A: Pure Mathematics [40 marks]**

- 1 (a) The graph of  $y = f(x)$  is shown below. The curve passes through the point  $A(-1,0)$ , the point  $B(1,0)$  and the point  $C(2,4)$ . The line  $x = 0$  is an asymptote of the curve.



Sketch the graph of  $y = \frac{1}{f(x)}$ , stating the equations of any asymptote(s), the coordinates of any point(s) of intersection with the axes and stationary point(s). [3]

- (b) The curve  $y = f(x)$  undergoes, in succession, the following transformations:

A: a translation of 4 units in the positive  $x$ -direction

B: a reflection in the  $y$ -axis

C: a stretch with scale factor 2 parallel to the  $x$ -axis

The equation of the resulting curve is  $y = \frac{x-1}{x+1}$ . Obtain the equation of the original curve

$y = f(x)$ .

[3]

- 2 (a) Find the volume generated when the area bounded by the curve  $y = \frac{x-1}{x+1}$ , the  $x$ -axis, and the lines  $x = 2$  and  $x = 3$  is rotated completely about the  $y$ -axis. Give your answer correct to 3 significant figures. [4]

- (b) A curve  $C$  is defined by the parametric equations

$$x = -\frac{1}{t}, \quad y = \frac{1}{\sqrt{4-t^2}}, \quad \text{where } -2 < t < 0.$$

The region  $S$  is bounded by  $C$ , the lines  $x = \frac{1}{\sqrt{3}}$ ,  $x = \frac{1}{\sqrt{2}}$  and the  $x$ -axis.

Show that the area of  $S$  can be expressed as  $\int_a^b \frac{1}{t^2\sqrt{4-t^2}} dt$ , where  $a$  and  $b$  are constants to be determined in exact form.

By using the substitution  $t = 2 \sin \theta$ , find the exact area of  $S$ . [6]

- 3 (a) Given that  $z = 1 + \sqrt{2}i$  is a root of the equation  
$$z^3 + az^2 + bz - 6 = 0,$$
find the values of the real numbers  $a$  and  $b$ . [4]
- (b) Show clearly on an Argand diagram the locus given by  $|z - a| \leq |a|$  where  $a = -\sqrt{2} + \sqrt{2}i$  [2]
- (i) Find exactly the least and maximum value of  $|z - 3 + \sqrt{2}|$ . [2]
- (ii) Find the range of values of  $\arg(z - 3 + \sqrt{2})$ . [4]

- 4 (a) Given that  $x^2 + 2x - 1$ ,  $x^2 - 3$ ,  $x^2 - 2x + 1$  are consecutive terms of a convergent geometric progression, find all the real value(s) of  $x$  and the common ratio of the geometric progression. [4]
- (b) A farmer intends to plant 201 saplings at 1 meter apart between consecutive saplings along one edge of a field AB, 200m long, to form a fence.

All the saplings are delivered at A and must be carried to their planting holes one at a time. The first sapling is planted at A. For example, the farmer plants the first sapling at A. Then he walks 1m to plant 2<sup>nd</sup> sapling and 1m back to A to pick up the next sapling. Then he walks 2m to plant the 3<sup>rd</sup> sapling and 2m back to A and so on. After the last sapling is planted, the farmer does not need to return to A.

Show that the farmer needs to walk a total of 40,000m in order to plant all the saplings. [3]

In trying to save walking distance, the farmer thought of another method: he carries two saplings at a time, plants the first at A and walks 1m to plant the second, before returning to A. The process is repeated for the remaining saplings. (Note that the last sapling has to be carried singly, as there is an odd number of saplings.) When he has planted the last plant, he does not need to return to A. How much walking distance would this method save the farmer? [5]

**Section B: Statistics [60 marks]**

- 5 A cosmetic company wants to select a sample of 400 members to participate in a survey. The company sent the survey forms to all its 3000 members and collated the responses of the first 200 working adults who are aged 22 years to 35 years and the first 200 working adults who are aged 36 years to 45 years.
- (i) What name is given to this type of sampling? [1]
  - (ii) Suggest one advantage of this sampling method. [1]
  - (iii) Suggest another method of sampling and state an advantage of this method over the method used by the cosmetic company. [2]

- 6 Mr and Mrs Tan have a son named David. They invite 6 guests to their home for dinner. Before the dinner, they plan to take a group photo together with the guests. They are to stand in a straight row for the photo-taking.

Find the number of arrangements for the photo-taking if:

- (i) all the 9 diners can stand in any position, [1]
- (ii) the Tan family members must be separated. [2]

The Tan family decides to seat the diners at two round tables – a five-seater and a smaller four-seater. Find the number of arrangements this can be done if:

- (iii) anyone can be seated at either table; [2]
- (iv) David must be seated between his parents at either table. [3]

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- 7 Cotton Empire sells cotton candies in packs with a mean weight of  $\mu_0$  and standard deviation  $\sigma$ . Their competitor, Sweet Stuff, suspects that Cotton Empire has changed the average weight of their packs of cotton candies. Sweet Stuff bought a random sample of 60 packs of cotton candies and the weight,  $x$  g, of each pack was recorded. The data was summarised by

$$\bar{x} = 35 \text{ and } \sum (x - 35)^2 = 180$$

(i) Find the unbiased estimate of the population variance. [1]

(ii) Find the range of values of  $\mu_0$  such that Sweet Stuff did not have sufficient evidence to justify their suspicion at the 5% level of significance. [4]

(iii) It is given that  $\mu_0$  is now known to be 35.7 g. Sweet Stuff took another sample of 5 packs with following weights (in g)

37, 35.2, 35.3, 34.5, 33

Combining these 5 packs with the sample of 60 packs earlier to form a sample of 65, test at 2% level of significance if Sweet Stuff's claim can be justified. [3]

- 8 A particular disease infects a proportion  $p$  of the population of a town. A medical test has been administered to the entire population, but unfortunately the test is not completely reliable. If an individual has the disease, there is a probability 0.98 of getting a positive result, and if an individual does not have the disease, there is a probability of 0.08 of getting a positive result. An individual is chosen at random and tested.

Using a probability tree, find in terms of  $p$ , the probability that

- (i) the individual has the disease given that the result is positive. [3]  
(ii) the test will lead to a wrong conclusion. [2]

It is decided that if the result of the test on someone is positive, that person will be tested again. The result of the second test is independent of the first test.

- (iii) Find the probability that the person has the disease given that the result of the second test is positive. [2]  
(iv) The town has a population of 12,000 people and the mayor plans to test all of the 12,000 people in the town. Assuming  $p = 0$  (i.e. no one in town is infected by the disease), calculate the expected number of people who need to take a second test. [2]

- 9 A particular type of LCD (liquid crystal display) screen contains a large number of pixels, each of which may fail to work properly.
- (i) State a condition for Poisson distribution to be a suitable distribution for the number of failed pixels in a randomly chosen LCD screen. [1]

You may assume that the number of failed pixels in a randomly chosen LCD screen follows a Poisson distribution with mean 0.2.

- (ii) Find the probability that there are a total of more than 3 failed pixels and at most 6 failed pixels in 10 randomly chosen LCD screens. [3]
- (iii) An LCD screen is considered “imperfect” if it contains any failed pixel. Find the probability that a randomly selected LCD screen is imperfect. Hence, by using a suitable approximation, find the probability that there are 10 imperfect LCD screens in a batch of 50. [4]
- (iv) The manufacturer sends out a batch of 50 LCD screens every week. Find the probability that the mean number of imperfect LCD screens sent out by the manufacturer per week in 52 weeks is more than 8. [2]

- 10 The operator of Queen Motorways records its weekly earnings from road toll charges according to the categories of vehicles using the road. The weekly earnings (in thousands of dollars) for each category are assumed to be normally distributed. These distributions are independent of one another and are summarised in the table below.

Vehicle Category	Mean (thousands)	Standard deviation (thousands)
Cars	120.3	10.4
Buses	69.2	12.5
Lorries	64.5	9.5

Find the probability that the difference in weekly earnings for buses and cars is not more than \$60,000. [3]

Find the probability that over a 5-week period, the total earnings for lorries exceed \$345,000. What assumption must be made in your calculation? [3]

Each week, the operator allocates part of the earnings for repairs. This is determined for each category of vehicle according to estimates of long-term damaged caused. It is calculated as follows:

$x\%$  of earnings from cars, 8% from buses and 15% from lorries.

Given that the probability that the total amount for repairs is at least \$25,000 in a given week is 0.097, find  $x$ . [3]

Queen Motorways also records its weekly takings from collection of administration fees from drivers who pay the toll charges by cash. The mean weekly takings is \$2000 and the standard deviation of the weekly takings is \$800. State, with a reason, whether the weekly takings follows a normal distribution. [1]

- 11 The table below showed the average healthcare expenditure per person yearly and their expected life expectancy of people living in ASEAN countries.

Country	Healthcare Expenditure (US\$), $h$	Life Expectancy (years), $l$
Brunei Darussalam	792	77
Cambodia	42	61
Indonesia	55	68
Lao People's Democratic Republic	36	63
Malaysia	337	73
Myanmar	12	64
Philippines	67	70
Singapore	1503	82
Thailand	168	70
Viet Nam	80	72

Data extracted from World Health Organisation (WHO) <http://apps.who.int/ghodata/>

- (i) Calculate the product moment correlation coefficient between  $h$  and  $l$ , and explain whether the answer suggests that a linear model is appropriate. [3]
- (ii) Draw a scatter diagram for the data. [1]
- (iii) Explain why the scatter diagram of the data may be consistent with a model of the form  $l = a + b \ln h$ . [1]
- (iv) Briefly explain, in context to the question, why a linear relationship between  $h$  and  $l$ , is not appropriate. [1]
- (v) Calculate the least squares estimates of  $a$  and  $b$  for the model  $l = a + b \ln h$ . [2]
- (vi) Using the model in (v), predict the value of  $l$  to the nearest year when  $h = 500$ . Comment on the validity of your prediction [2]
- (vii) A rich tycoon in Indonesia saw the data and claimed that by spending US\$1503 yearly in Indonesia, his life expectancy will be 82 years. Comment on the validity of his statement. [1]