CIRCULAR MOTION

Challenging **MCQ** questions by The Physics Cafe

Compiled and selected by The Physics Cafe

1 A small cube of mass m slides down along a spiral path round a cone as shown in Fig 2.1. There is a smooth wall along the outer edge of the spiral path to prevent the cube from falling out of the path. This wall is inclined such that it always exerts a horizontal contact force on the cube as it spirals down. The path is always inclined at an angle θ to the horizontal at any point as shown in Fig. 2.2. All frictional forces are negligible.

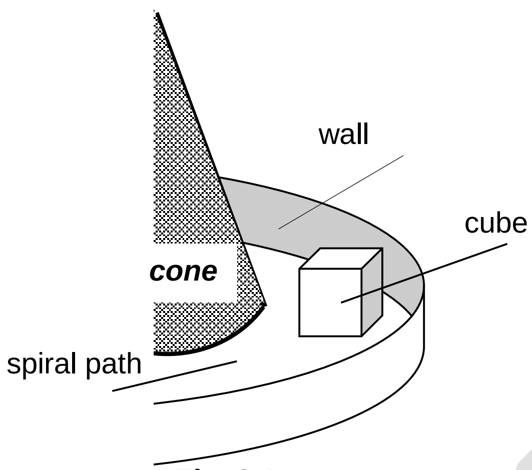

Fig. 2.1

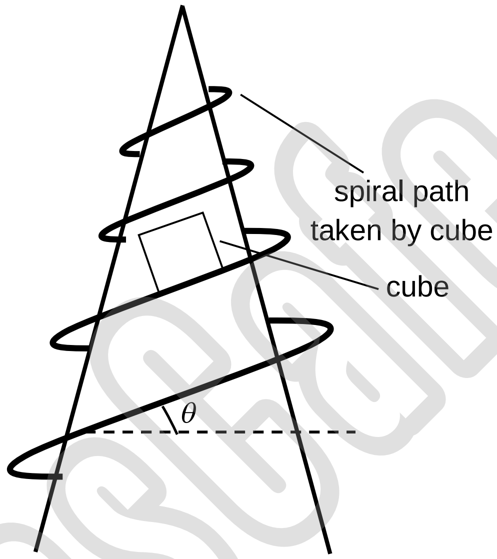

Fig. 2.2

(a) There are three distinct forces acting on the cube, including the horizontal contact force.
Fig. 2.3 is a zoomed-in view of the cube when it is at the position in Fig. 2.2. In Fig. 2.3, draw the forces that act on the cube, paying particular attention to the point of application of each force. Your forces should be clearly labelled in words, describing the nature of each force. [3]

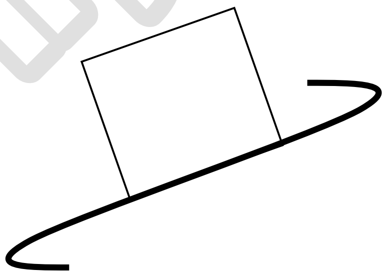

Fig. 2.3

(b) Based on your answer in **(a)**, explain how the forces affect the motion of the cube as it slides down the spiral path.

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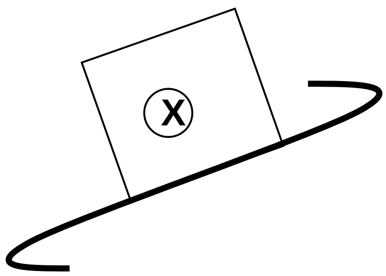
………………………………………………………………………………………………… [2]

(c) Derive an expression for the rate of change of kinetic energy of this cube in terms of θ, *m*, acceleration of free fall *g*, and its instantaneous speed *v*. [2]

Ans (a)

Direction of the Normal Contact force from wall should be drawn into the page using the right symbol.

Otherwise, a clear statement should be given to describe its direction.



B1 x 3 – for each correct force

(b) Normal horizontal contact force of wall on cube provides the centripetal force for cube to move in a circular path as it spirals downward. B1

Component of the weight parallel to the slope causes the acceleration of cube down slope. [2] B1

(c) Rate of increase of kinetic energy

= rate of decrease of gravitational potential energy M1

= $mgv\sin\theta$ A1

OR

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{\frac{1}{2}m\left(v^2-u^2\right)}{t} = \frac{\frac{1}{2}m\left(2as\right)}{t} = \frac{\frac{1}{2}m\left(2g\sin\theta\, s\right)}{t}$$

where $a = g\sin\theta$ and $\frac{s}{t} = v$

OR

$$KE = \frac{1}{2}mv^2$$

$$\frac{\mathrm{d}KE}{\mathrm{d}t} = \frac{1}{2}m\frac{\mathrm{d}v^2}{\mathrm{d}t} = \frac{1}{2}m\,2v\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{2}m\,2va$$

where $a = g\sin\theta$

2 At the National Day parade, a parade baton is twirled by a member of the marching contingent. The baton can be modelled to be a small ball of mass *m* fixed to one end of a light rigid rod. During the twirl, the ball can be assumed to move at constant speed around the circumference of a vertical circle with centre at *C*, as shown in Fig. 3.1.

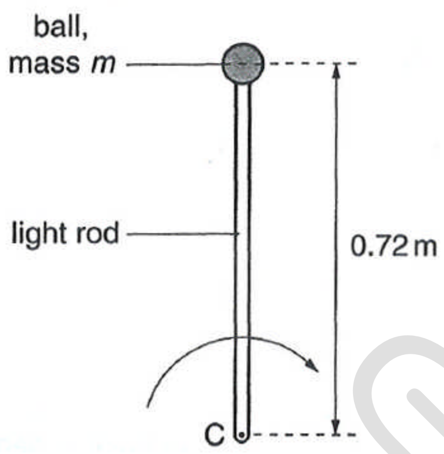


Fig. 3.1

(a) Explain what is meant by *centripetal force*.

………

………

……… [2]

(b) When the rod is vertical with the ball above point *C*, the tension *T* in the rod is

T = 2mg

where g is the acceleration of free fall.

(i) State in terms of *mg*, the magnitude of the centripetal force.

centripetal force = …………………………….. [1]

(ii) Determine the magnitude of the tension, in terms of *mg,* in the rod when it is vertical, with the ball below point *C*.

tension = …………………………….. [1]

(iii) Determine the angular speed of the ball.

angular speed = …………………………….. rad s^{-1} [2]

(iv) Given that the ball moves with a constant angular speed, explain why work has to be done for it to move from the position where it is vertically above point C to the position where it is vertically below C.

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…………………………………………………………………………………………[1]

Ans (a) Centripetal force is the force needed to maintain a body in circular motion, and is provided by the resultant force acting on the body. B1

Its direction is always perpendicular to the velocity of the body, directed toward the center of curvature of the path. B1

(b) (i) 3 mg A1

(ii) 4 mg A1

(iii) At top, $T + W = F_c$

$3\ mg = m\ r\ \omega^2$ M1

$\omega = \sqrt{\frac{3g}{r}} = 6.4$ rad s^{-1} A1

(iv) By conservation of energy, the ball loses GPE as it moves downwards, which may be converted to KE. Hence, in order to move the ball at constant angular speed, work has to be done to convert the GPE (which it would otherwise gain as KE when it moves from position above C to below C) to other forms. B1

3 In the bagatelle board shown in **Fig. 4**, energy stored in a compressed spring is released to project a ball of mass **50 g** along the surface of the board, which has a raised end and is inclined at **20⁰** to the horizontal.

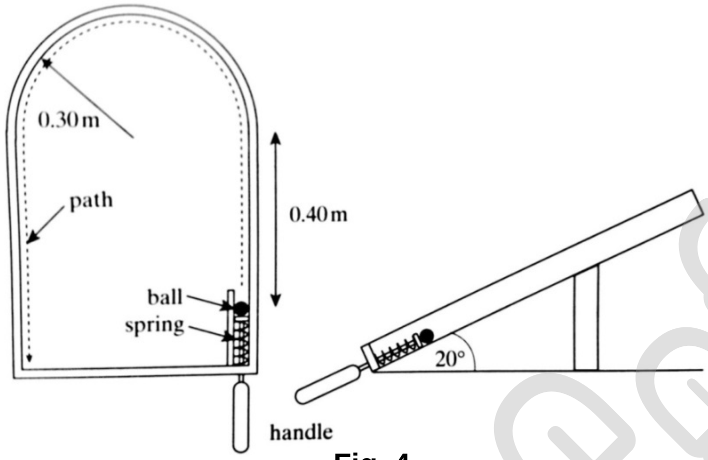


Fig. 4

The diagram shows the ball resting on the relaxed spring. Pulling on the handle with a force of **6.15 N** compresses the spring by **50 mm** and when the handle is released, the ball follows a path marked by the dotted line. Assume that frictional losses and the kinetic energy of the handle and spring are negligible. Ignore the effect of rotation.

(a) (i) Calculate the speed of the ball as it reaches the top of the curve section.

[2]

(ii) Draw a free body diagram showing all the forces acting on the ball at the instance in

[2]

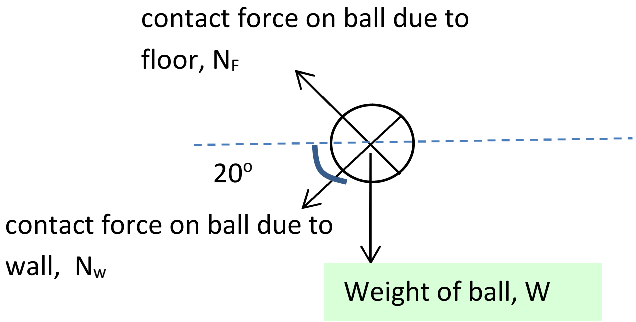
(b) Show that the speed calculated in part 4**(a)(i)** is just sufficient for the ball to complete the path shown, explaining your reasoning.

[3]

Ans ai Applying the Principle of Conservation of Energy on the energy of the ball:
Loss in elastic potential energy = Gain in kinetic energy + gain in gravitational potential energy
½ Fx = ½ mv^2 + mgh
½ (6.15) (50x10^{-3}) = ½ (0.050)v^2 + (0.050)(9.81)(50x10^{-3} + 0.70)sin 20^o
v = 1.057
= 1.06 ms^{-1}

[2]

aii



[2]

b For the ball to complete the circular motion, the centripetal force is provided by the contact force on ball due to wall and component of the weight that is directed to the centre of the circle.

Using Newton's 2nd law: Fnet = ma
N_w+ Wsin20^o = mv^2/r
N_w = mv^2/r - Wsin20^o
For circular motion to be completed, N_w>0
mv^2/r - Wsin20^o > 0
mv^2/r > mgsin20^o
v > √(rgsin20^o)
v >1.003 ms^{-1}

Since the speed in (a)(i) is greater than the minimum speed required, the ball is able to complete the circular path.